ABSTRAKSI & GENERALISASI

Sebagai Salah Satu Alat Untuk Memunculkan *Conjecture* Dalam Penelitian Matematika (Khususnya Bidang Aljabar)

Disampaikan Pada Kuliah Tamu Bidang Aljabar

Jurusan Matematika FMIPA Unand

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Abstrak

- Abstraksi dan generalisasi pada matematika merupakan **proses yang dimulai dengan contoh konsep dan prosedur yang dipandang sudah familiar.**
- Dengan tujuan akhir untuk mengetahui **apakah konsep dan prosedur ini bersifat umum – yang diharapkan akan berguna untuk mengetahui situasi yang berbeda termasuk situasi yang belum pernah dihadapi.**
- Dalam presentasi ini, akan ditampilkan beberapa **contoh proses abstraksi dan generalisasi yang menginspirasi munculnya topik-topik dalam matematika untuk dapat dikembangkan sebagai penelitian bidang matematika khususnya bidang penelitian aljabar.**
- Contoh-contoh yang ditampilkan merupakan **fenomena yang sudah sangat familiar dengan keseharian bermatematika dengan harapan dapat untuk model dalam memunculkan dugaan (*conjecture*) dalam penelitian matematika.**

Latar Belakang:
Pemilihan Topik (**Abstraksi & Generalisasi**)

- Kurikulum Berbasis Kompetensi Mengacu Pada Ketentuan KKNI, yang mana PS harus merumuskan **Capaian Pembelajarannya.**
    - **Rekomendasi IndoMS** tentang Capaian Pembelajaran
    - Rumusan **Learning Outcome** (Capaian Pembelajaran) **Lembaga Akreditasi Internasional ASIIN**
    - Benchmark rumusan Learning Outcome (Capaian Pembejaran) beberapa Program Studi Matematika di LN (diantaranya **University of California Santa Cruz**)
    - Dll

Salah Satu Rumusan Capaian Pembelajaran Yang Telah Disepakati oleh IndoMS

- Lulusan Matematika yang mampu mengembangkan pemikiran matematis, yang diawali dari pemahaman prosedural / komputasi hingga pemahaman yang luas meliputi eksplorasi, penalaran logis, generalisasi , abstraksi , dan bukti formal

Learning Outcomes Mathematics Program (ASIIN – Lembaga Akreditasi Internasional)

https://www.asiin.de/en/quality-management/accreditation-degree-programmes.html

a. Specialist learning outcomes

Graduates

- have sound mathematical knowledge. They have a profound overview of the contents of fundamental mathematical disciplines and are able to identify their correlations.
- are able to recognise mathematics-related problems, assess their solvability and solve them within a specified time frame.
- have a basic ability to work in a scientific way. They are in particular able to formulate mathematical hypotheses and have an understanding of how such hypotheses can be verified or falsified using mathematical methods.
- can flexibly apply mathematical methods of fundamental component areas of mathematics and are able to transfer the findings obtained to other component areas or applications.
- have abstraction ability and are able to recognise analogies and basic patterns.
- are able to think in a conceptual, analytical and logical manner.
- have an extensive comprehension of the significance of mathematical modelling. Are able to create mathematical models for mathematical problems as well as for problems in other areas of science or everyday life, and have a selection of problem solving strategies at their disposal.

have abstraction ability and are able to recognise analogies and basic patterns.

Mathematics Undergraduate Student Learning Objectives

(Contoh University of California Santa Cruz)

https://www.math.ucsc.edu/undergraduate/undergrad_learn_outcomes.html

- The Mathematics program promotes mathematical skills and knowledge for their intrinsic beauty, effectiveness in developing proficiency in analytical reasoning, and utility in modeling and solving real world problems.
- To responsibly live within and participate in the transformation of a rapidly changing, complex, and interdependent society, students must develop and unceasingly exercise their analytical abilities.
- Students who have learned to logically question assertions, recognize patterns, and distinguish the essential and irrelevant aspects of problems can think deeply and precisely, nurture the products of their imagination to fruition in reality, and share their ideas and insights while seeking and benefiting from the knowledge and insights of others.

Students majoring in Mathematics attain proficiency in

- *Critical thinking*
  - The ability to identify, reflect upon, evaluate, integrate, and apply different types of in- formation and knowledge to form independent judgments. Analytical and logical thinking and the habit of drawing conclusions based on quantitative information.
- *Problem solving*
  - The ability to assess and interpret complex situations, choose among several potentially appropriate mathematical methods of solution, persist in the face of difficulty, and present full and cogent solutions that include appropriate justification for their reasoning.
- *Effective communication*
  - The ability to communicate and interact effectively with different audiences, developing their ability to collaborate intellectually and creatively in diverse contexts, and to appre- ciate ambiguity and nuance, while emphasizing the importance of clarity and precision in communication and reasoning.

Critical thinking

- Students will
  - understand the basic rules of logic, including the role of axioms or assumptions
  - appreciate the role of mathematical proof in formal deductive reasoning
  - be able to distinguish a coherent argument from a fallacious one, both in mathematical reasoning and in everyday life
  - understand and be able to articulate the differences between inductive and deductive reasoning
  - proficiently construct logical arguments and rigorous proofs
  - **formulate conjectures by abstracting general principles from examples.**

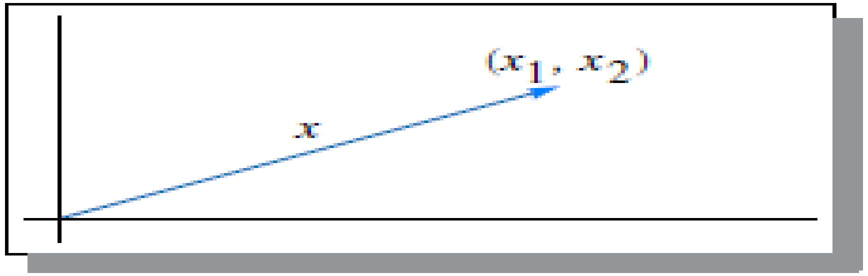
Beberapa Contoh Abstraksi dan Generalisasi
(Pembejalaran Struktur Aljabar dan Aljabar Linear)

- **The Power of Himpunan Bilangan Bulat Z**
- **Konsep Inner Product (Hasil Kali Dalam) Dalam Aljabar Linear**

Vektor
(**Besaran Yang Mempunyai Besar dan Arah**)

6.A *Inner Products and Norms*

Inner Products



The length of this vector x is $\sqrt{x_1^2 + x_2^2}$.

To motivate the concept of inner product, think of vectors in $\mathbf{R}^2$ and $\mathbf{R}^3$ as arrows with initial point at the origin. The length of a vector x in $\mathbf{R}^2$ or $\mathbf{R}^3$ is called the ***norm*** of x, denoted $\|x\|$. Thus for $x = (x_1, x_2) \in \mathbf{R}^2$, we have $\|x\| = \sqrt{x_1^2 + x_2^2}$.

Similarly, if $x = (x_1, x_2, x_3) \in \mathbf{R}^3$, then $\|x\| = \sqrt{x_1^2 + x_2^2 + x_3^2}$.

Even though we cannot draw pictures in higher dimensions, the generalization to $\mathbf{R}^n$ is obvious: we define the norm of $x = (x_1, \ldots, x_n) \in \mathbf{R}^n$ by

$$\|x\| = \sqrt{x_1^2 + \cdots + x_n^2}.$$

6.2 Definition ***dot product***

For $x, y \in \mathbf{R}^n$, the *dot product* of x and y, denoted $x \cdot y$, is defined by

$$x \cdot y = x_1 y_1 + \cdots + x_n y_n,$$

where $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$.

If we think of vectors as points instead of arrows, then $\|x\|$ should be interpreted as the distance from the origin to the point x.

Note that the dot product of two vectors in $\mathbf{R}^n$ is a number, not a vector. Obviously $x \cdot x = \|x\|^2$ for all $x \in \mathbf{R}^n$. The dot product on $\mathbf{R}^n$ has the following properties:

- $x \cdot x \geq 0$ for all $x \in \mathbf{R}^n$;
- $x \cdot x = 0$ if and only if $x = 0$;
- for $y \in \mathbf{R}^n$ fixed, the map from $\mathbf{R}^n$ to $\mathbf{R}$ that sends $x \in \mathbf{R}^n$ to $x \cdot y$ is linear;
- $x \cdot y = y \cdot x$ for all $x, y \in \mathbf{R}^n$.

6.3 Definition *inner product*

An *inner product* on V is a function that takes each ordered pair (u, v) of elements of V to a number $\langle u, v \rangle \in \mathbf{F}$ and has the following properties:

positivity

$\langle v, v \rangle \geq 0$ for all $v \in V$;

definiteness

$\langle v, v \rangle = 0$ if and only if $v = 0$;

additivity in first slot

$\langle u + v, w \rangle = \langle u, w \rangle + \langle v, w \rangle$ for all $u, v, w \in V$;

homogeneity in first slot

$\langle \lambda u, v \rangle = \lambda \langle u, v \rangle$ for all $\lambda \in \mathbf{F}$ and all $u, v \in V$;

conjugate symmetry

$\langle u, v \rangle = \overline{\langle v, u \rangle}$ for all $u, v \in V$.

6.4 Example ***inner products***

(a) The *Euclidean inner product* on $\mathbf{F}^n$ is defined by

$$\langle (w_1, \ldots, w_n), (z_1, \ldots, z_n) \rangle = w_1\overline{z_1} + \cdots + w_n\overline{z_n}.$$

(b) If $c_1, \ldots, c_n$ are positive numbers, then an inner product can be defined on $\mathbf{F}^n$ by

$$\langle (w_1, \ldots, w_n), (z_1, \ldots, z_n) \rangle = c_1 w_1\overline{z_1} + \cdots + c_n w_n\overline{z_n}.$$

(c) An inner product can be defined on the vector space of continuous real-valued functions on the interval $[-1, 1]$ by

$$\langle f, g \rangle = \int_{-1}^{1} f(x)g(x)\,dx.$$

(d) An inner product can be defined on $\mathcal{P}(\mathbf{R})$ by

$$\langle p, q \rangle = \int_{0}^{\infty} p(x)q(x)e^{-x}\,dx.$$

Norms

Our motivation for defining inner products came initially from the norms of vectors on $\mathbf{R}^2$ and $\mathbf{R}^3$. Now we see that each inner product determines a norm.

6.8 Definition ***norm,*** $\|v\|$

For $v \in V$, the ***norm*** of v, denoted $\|v\|$, is defined by

$$\|v\| = \sqrt{\langle v, v\rangle}.$$

6.9 Example ***norms***

(a) If $(z_1, \ldots, z_n) \in \mathbf{F}^n$ (with the Euclidean inner product), then

$$\|(z_1, \ldots, z_n)\| = \sqrt{|z_1|^2 + \cdots + |z_n|^2}.$$

(b) In the vector space of continuous real-valued functions on $[-1, 1]$ [with inner product given as in part (c) of 6.4], we have

$$\|f\| = \sqrt{\int_{-1}^{1} (f(x))^2 \, dx}.$$

Sudut antara 2 vektor

6.15 Cauchy–Schwarz Inequality

Suppose $u, v \in V$. Then

$$|\langle u, v\rangle| \leq \|u\| \, \|v\|.$$

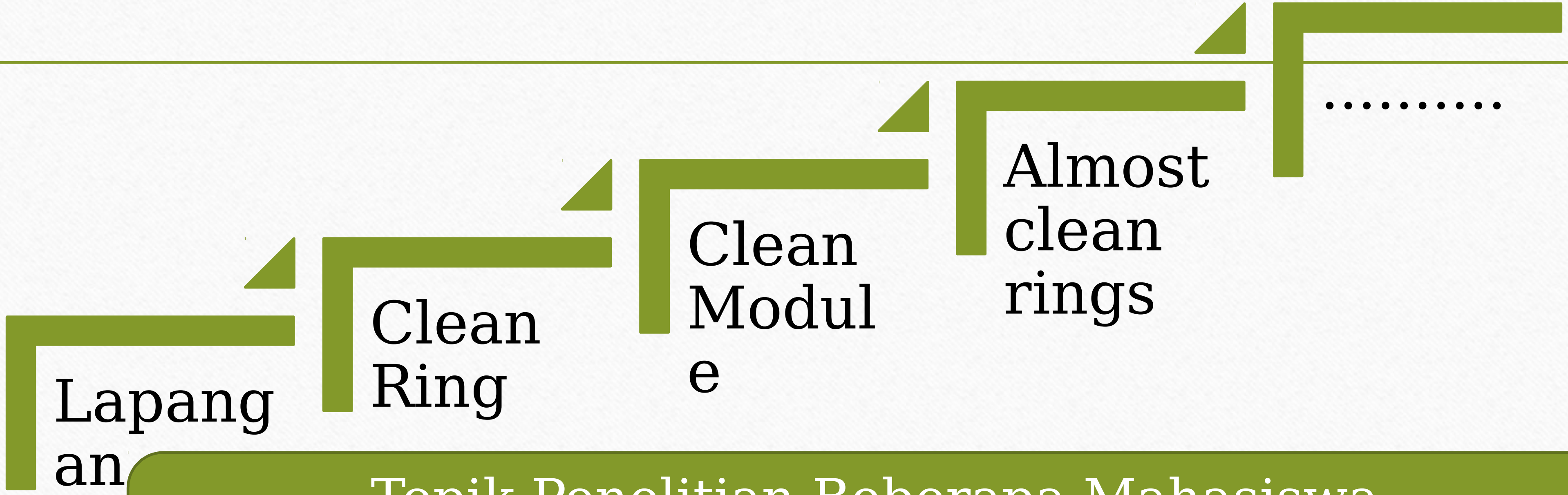
This inequality is an equality if and only if one of u, v is a scalar multiple of the other.

Kaitannya Dengan Topik-Topik Aljabar

Tugas Akhir Skripsi S1
Thesis S2
Disertasi S3

Beberapa Contoh Abstraksi dan Generalisasi

(Memunculkan Konsep / Problem / Conjecture)



Topik Penelitian Beberapa Mahasiswa
S1 (Skripsi), S2 (thesis) dan
Mahasiswa S3 (Disertasi Sdr.Atun Ismawarti)

Beberapa Contoh Abstraksi dan Generalisasi

(Memunculkan Konsep / Problem / Conjecture)

Topik Penelitian Hibah Pascasarjana
Yang Melibatkan Beberapa Mahasiswa S2 dan
Mahasiswa S3 (Naimah Hijriati)

Diskusi dan Pertanyaan

Mahasiswa Yang Berani Bertanya Akan Dapat Hadiah (Bukan HP, Bukan Ipad, tapi ....)

Terimakasih
Semoga ***Sharing*** Kami Bermanfaat