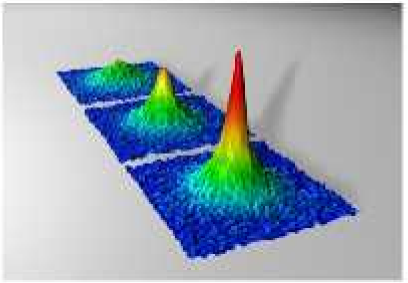
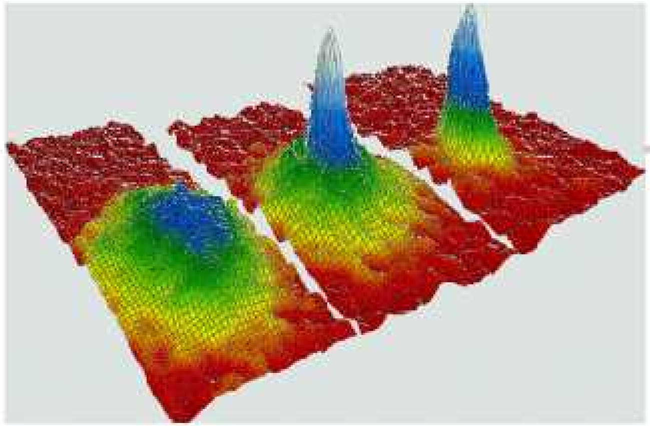


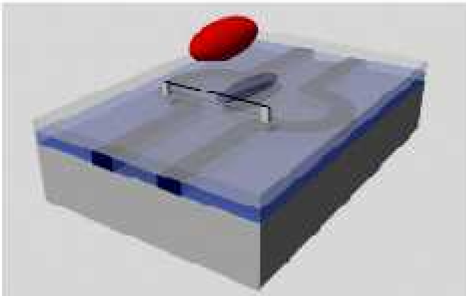
Manipulation of Ultracold Atoms



near semiconductor surfaces

Workshop "Introduction to Solitary Waves: Mathematical & Physical Perspectives"

26th February 2018 (Monday)



SEMINAR ROOM
DEPARTMENT OF MATHEMATICS
ANDALAS UNIVERSITY

Dr. Mohamad Nazri Abdul Halif
Institute of Engineering Mathematics
UNIVERSITI MALAYSIA PERLIS (UniMAP)
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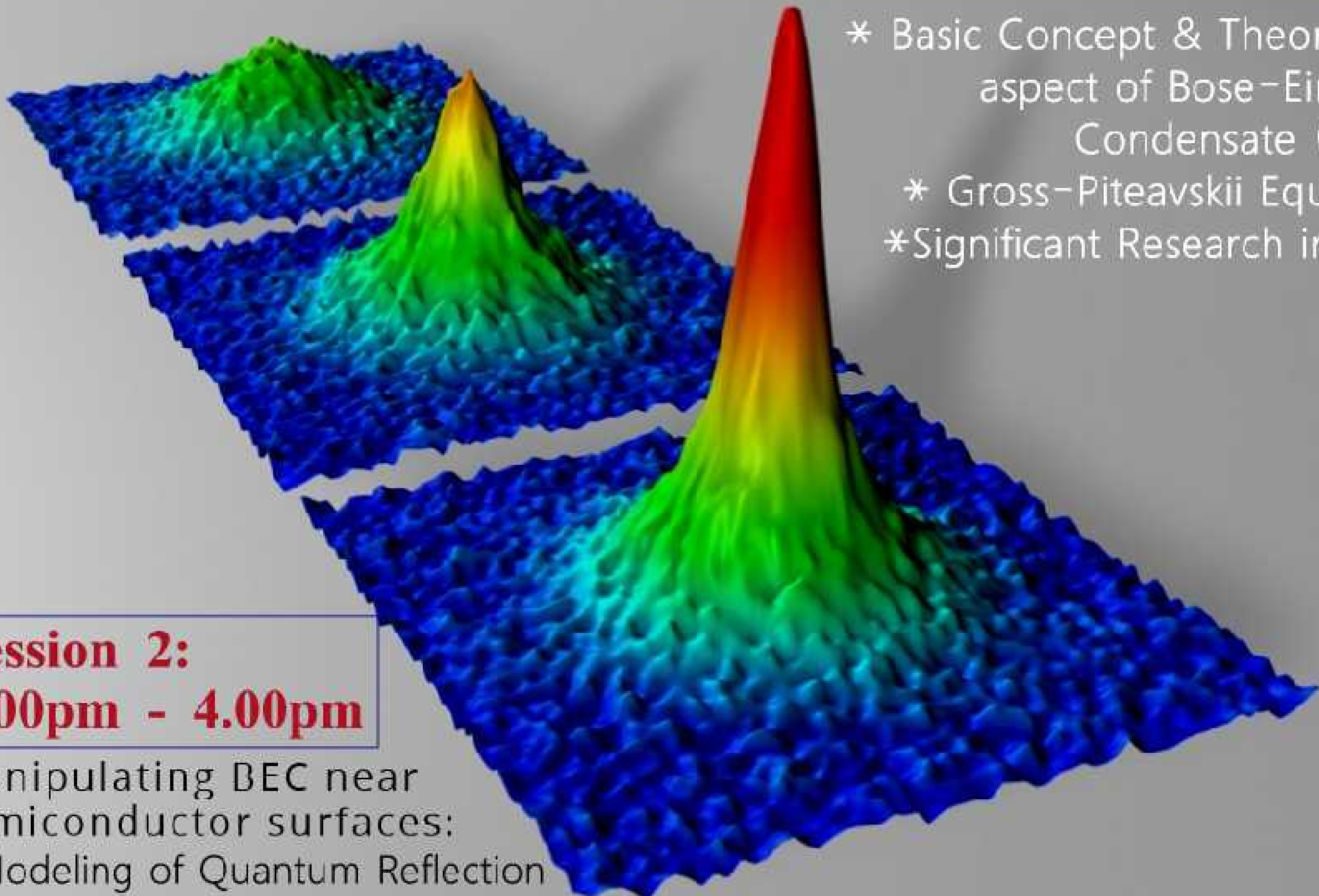
UniMAP UNIVERSITI MALAYSIA PERLIS

UNIVERSITAS ANDALAS
UNTUK KEDJAJAAN BANGSA

Content:

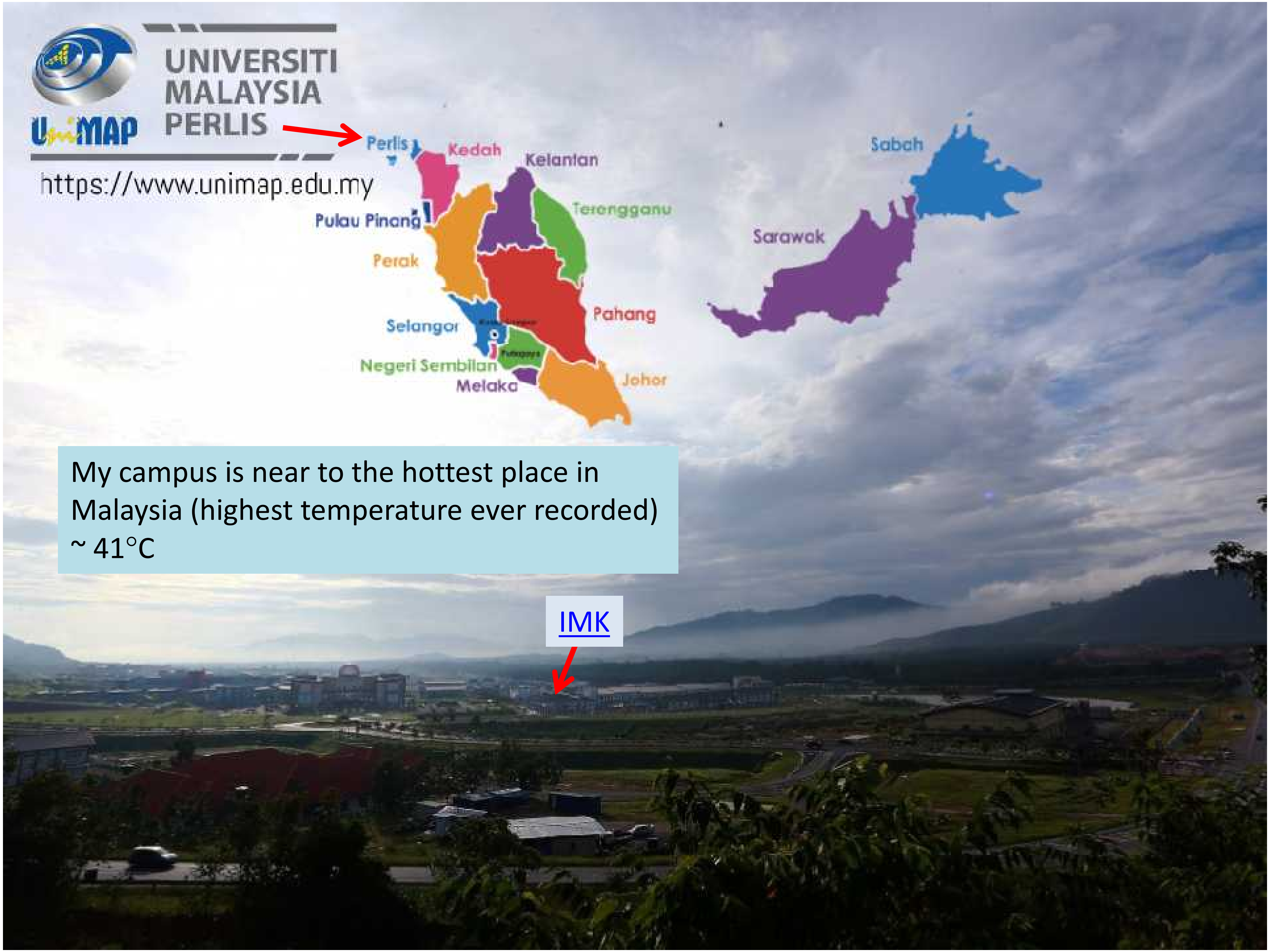
Session 1: 9.00am - 12.00pm

* Basic Concept & Theoretical aspect of Bose-Einstein Condensate (BEC)
* Gross-Piteavskii Equation
* Significant Research in BEC



Session 2:
2.00pm - 4.00pm

Manipulating BEC near semiconductor surfaces:
- Modeling of Quantum Reflection
- Modeling of atom-surface Casimir-Polder



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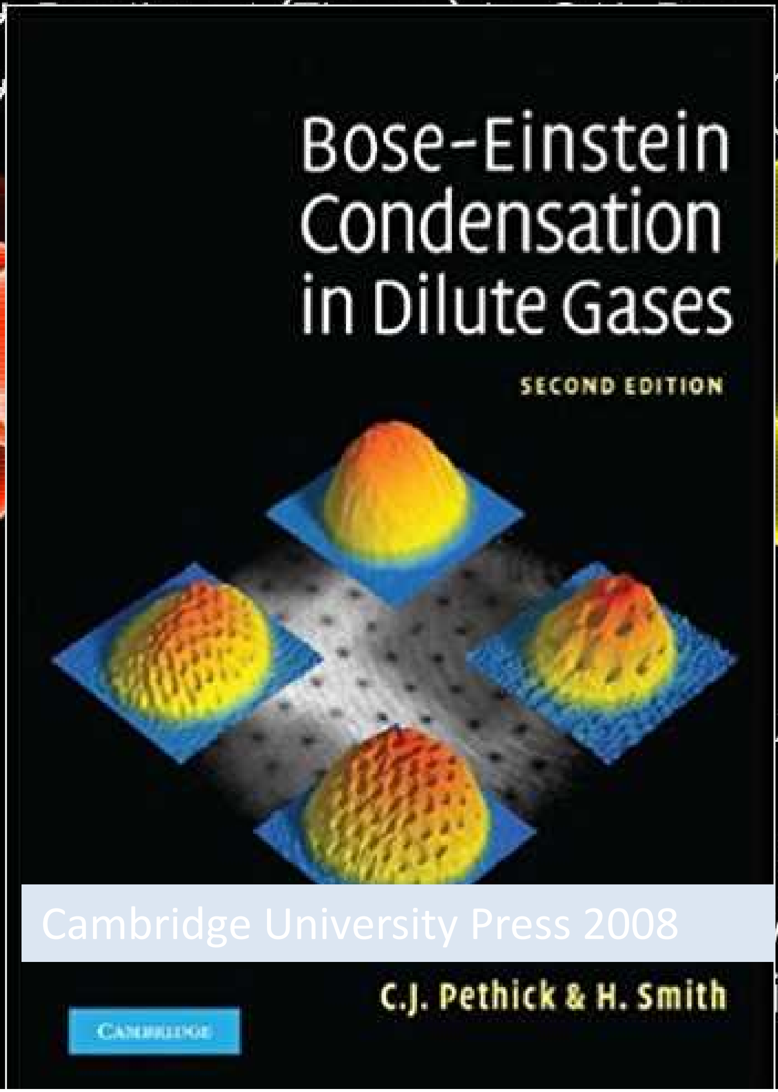
Terengganu
Pulau Pinang
Perak
Pahang
Selangor
Negeri Sembilan
Melaka
Johor
Sabah
Sarawak
My campus is near to the hottest place in Malaysia (highest temperature ever recorded) ~ 41°C
IMK

Bose-Einstein Condensate (BEC)

* Fifth (5th) Element of Matter (Theory: at 0 Kelvin (- 273.15 Celcius), Exp: at nano/pico Kelvin)

Coldest place in the universe! (~ nano Kelvin)



* & A. Einstein ~ 1924/1925
* (Awarded Nobel Prize in Physics 2001)

uantum Physics (Theoretical &

ience & Tech, Quantum Sensors, Atom
Atomic Clock, Foundation of Quantum
d systems, Quantum Engineering, etc...

Wave-particle duality

Overlap each other

Wave dominant

s-wave scattering

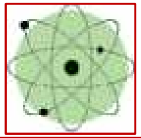
$\Delta x \Delta p \geq \frac{\hbar}{2}$

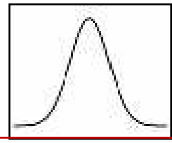
Few billionth of degree above absolute zero

Pico, Nano Kelvins

Speed of atoms are slowing down

$$a_{dB} = \sqrt{\frac{2\circledR^{2}\hbar^{2}}{m k_B T}}$$

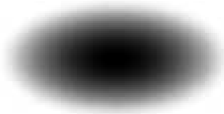




Wave-particle duality

PLASMA

$\approx > 6000$ K



≈ 0 K

Solid, Liquid, Gas

Room temperature 300K

5th state of matter:
ULTRACOLD ATOMS
(Bose-Einstein Condensates)

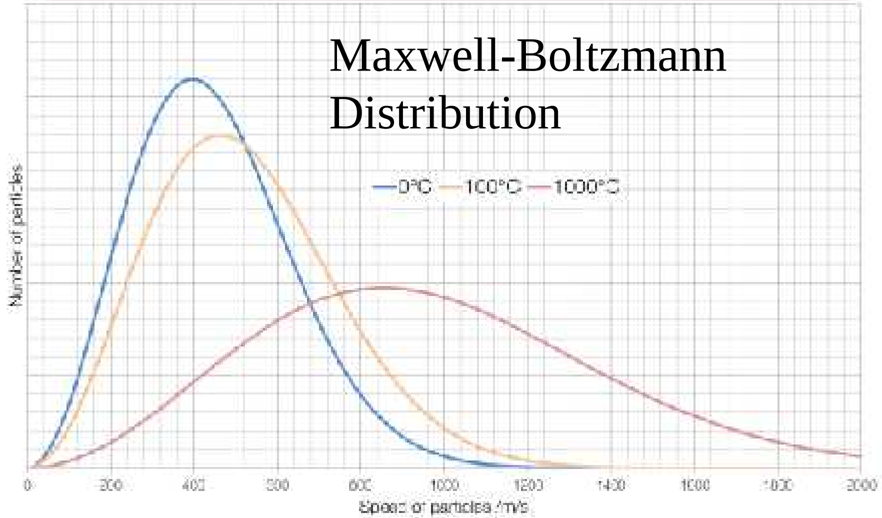
Atoms occupying the same space $|0\rangle$ & start to merge into one GIANT wave

Quantum world, view in macroscopic level

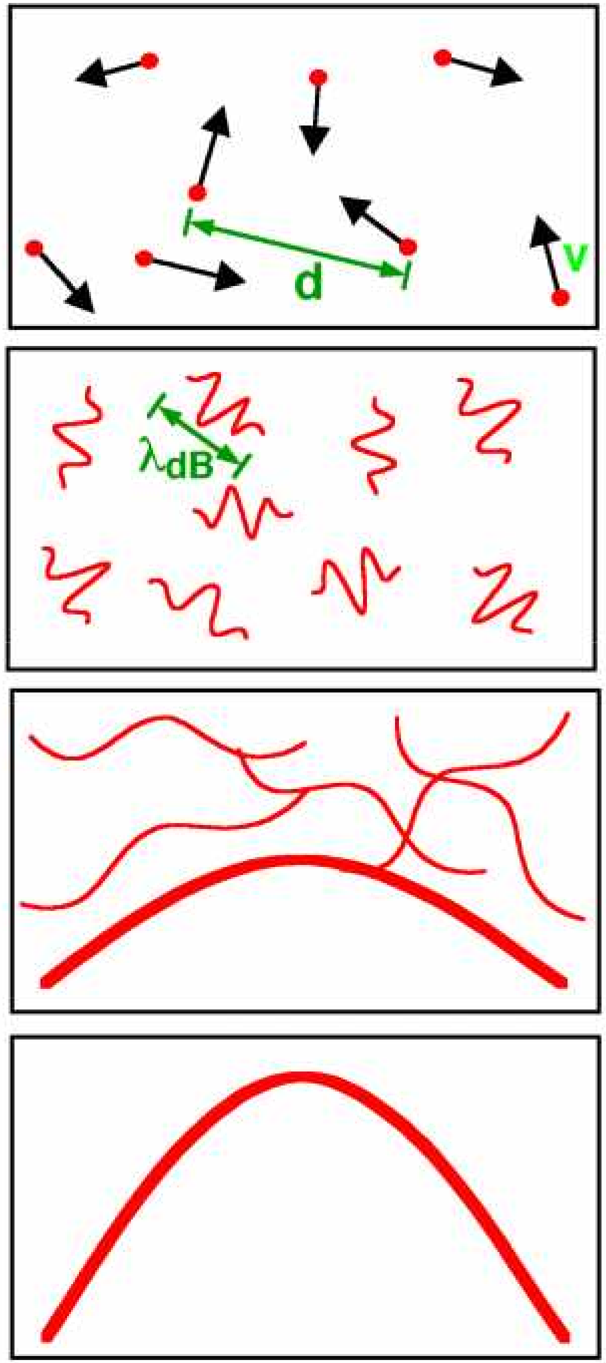
Fragile

0 K = 0 K.E (Classical)
0 K ≠ 0 K.E (Quantum)

Kinetic Energy (K.E) ==> Temperature



What is Bose-Einstein condensation (BEC)?



High Temperature T:
thermal velocity v
density d^{-3}
"Billiard balls"

Low Temperature T:
De Broglie wavelength
$\lambda_{dB} = h/mv \propto T^{-1/2}$
"Wave packets"

$T = T_{crit}$:
Bose-Einstein Condensation
$\lambda_{dB} \approx d$
"Matter wave overlap"

T=0:
Pure Bose condensate
"Giant matter wave"

Partic
de Bro
Identical
& Pauli's
Statistic
- Bose-E

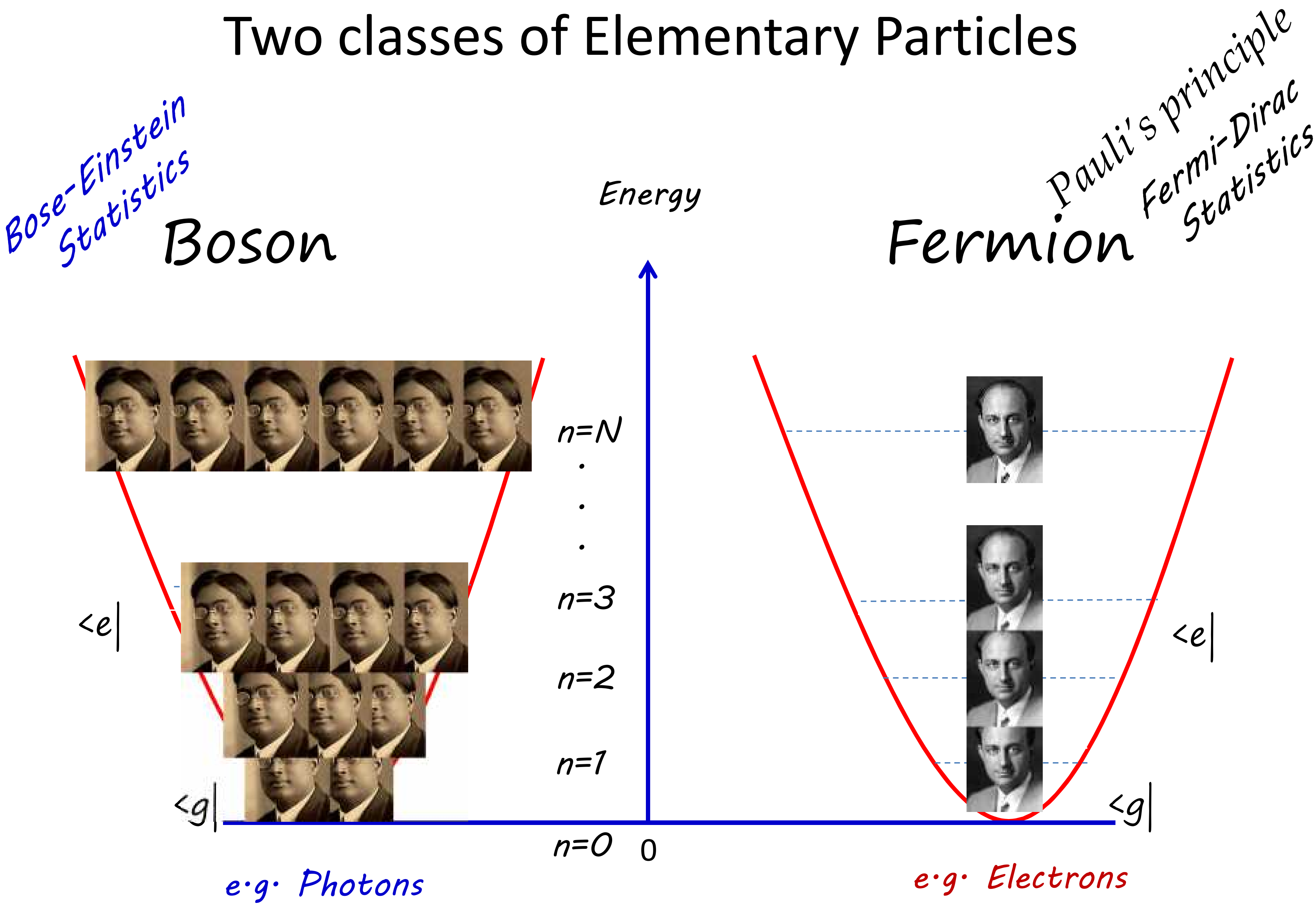
video1

EXTR
EXPERIM
Laser Co
Magnetic
Evaporat

ns:
ar BEC
nt Wave
nlinear)

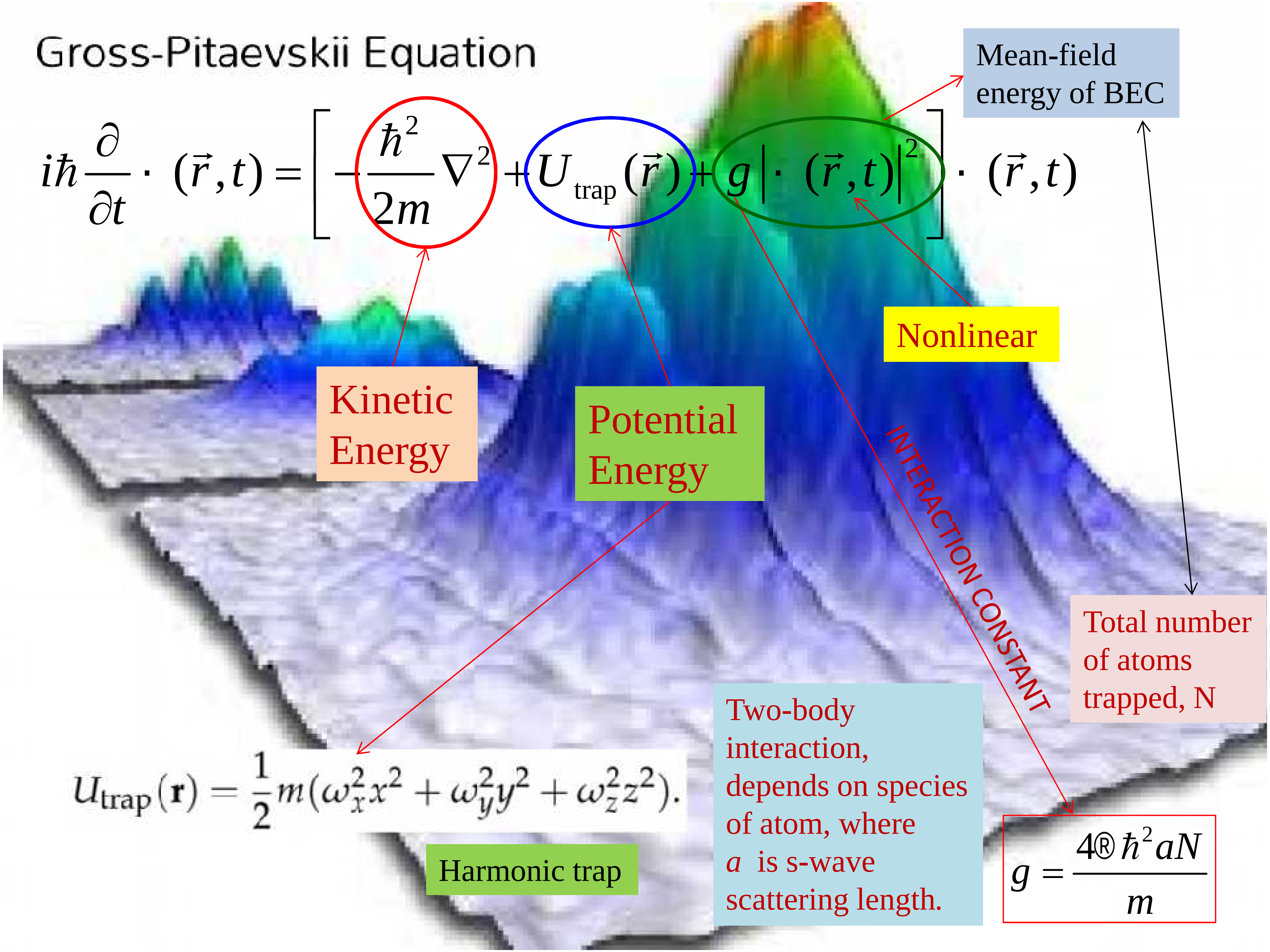
$+ |1\rangle$
$\sqrt{2}$

Two classes of Elementary Particles



Gross-Pitaevskii Equation

$$i\hbar \frac{\partial}{\partial t} \cdot (\vec{r},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + U_{\text{trap}}(\vec{r}) + g\left| \cdot (\vec{r},t)\right|^2 \right] \cdot (\vec{r},t)$$



Mean-field energy of BEC

Kinetic Energy

Potential Energy

Nonlinear

INTERACTION CONSTANT

Total number of atoms trapped, N

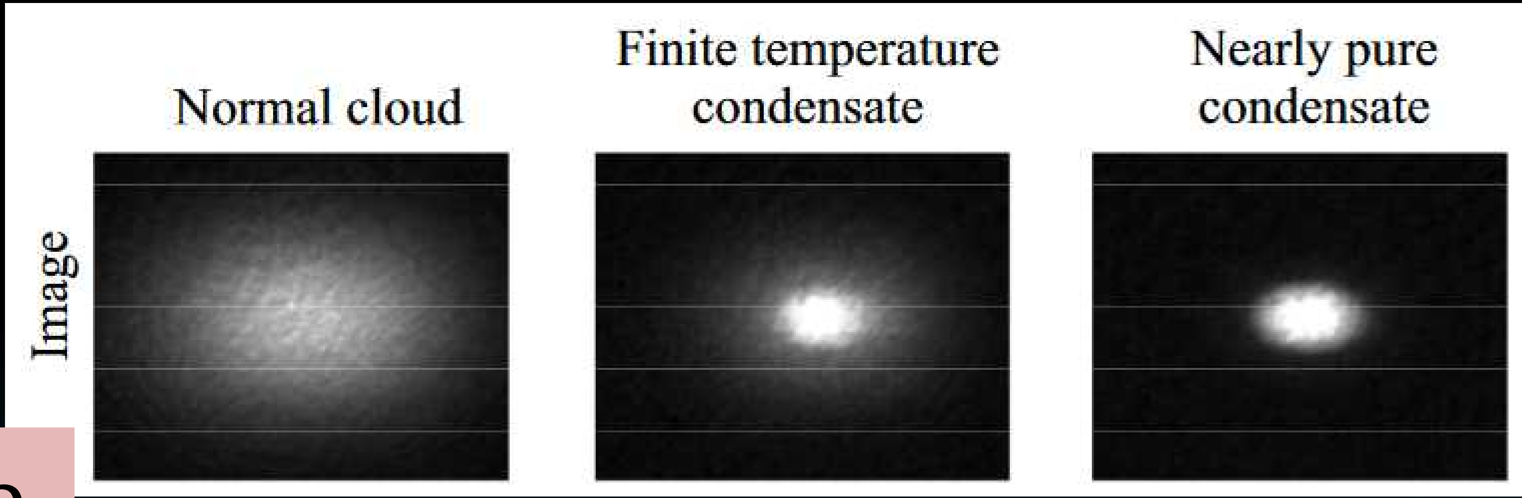
$$U_{\text{trap}}(\mathbf{r}) = \frac{1}{2}m(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2).$$

Harmonic trap

Two-body interaction, depends on species of atom, where a is s-wave scattering length.

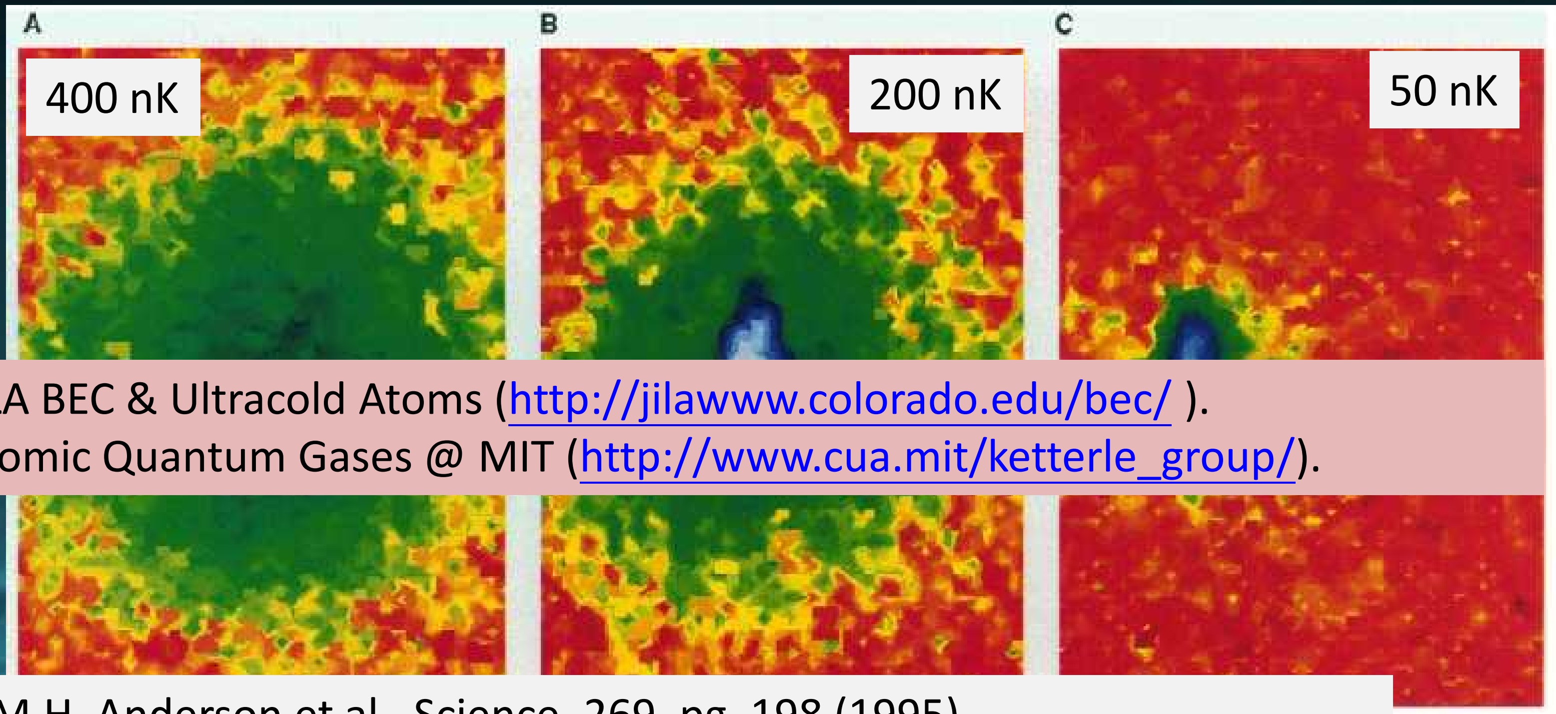
$$g = \frac{4\text{®}\,\hbar^2 a N}{m}$$

http://jila.colorado.edu/bec/CornellGroup/JLTP_Lewandowski2003.pdf



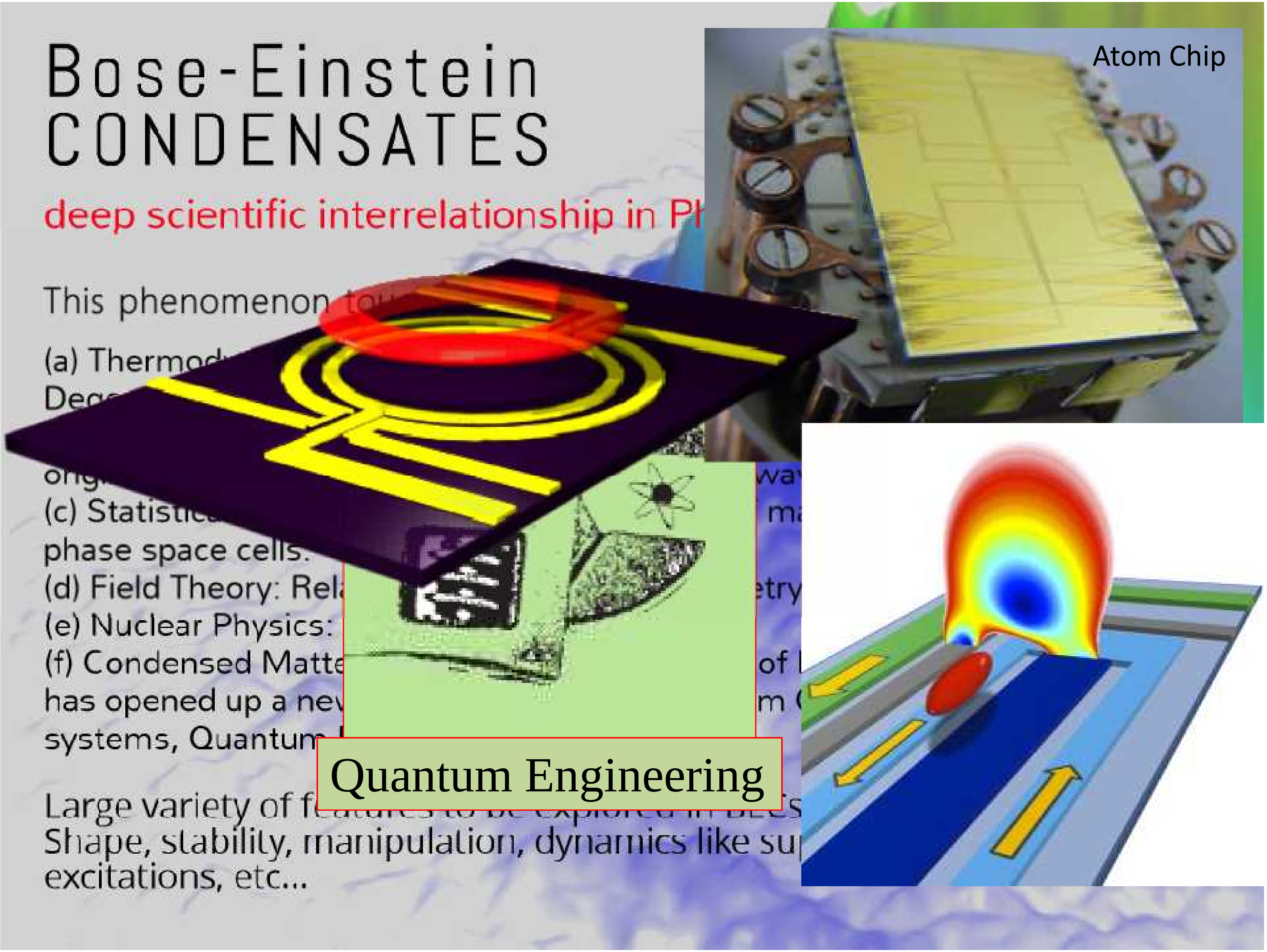
^{87}Rb

T ~ 313 nK **T ~ 225 nK** **T ~ 83 nK**



- ✱ JILA BEC & Ultracold Atoms (http://jilawww.colorado.edu/bec/).
- ✱ Atomic Quantum Gases @ MIT (http://www.cua.mit/ketterle_group/).

M.H. Anderson et al., Science, 269, pg. 198 (1995)



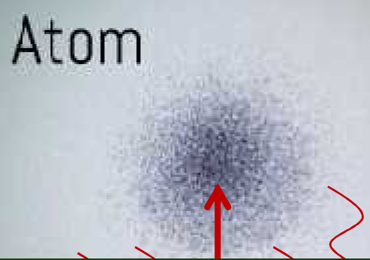
Bose-Einstein
CONDENSATES
deep scientific interrelationship in Ph
This phenomenon
phase space cells.
(d) Field Theory: Rel
(e) Nuclear Physics:
has opened up a ne
systems, Quantum
Large variety of f
Shape, stability, manipulation, dynamics like su
excitations, etc...
Atom Chip
Quantum Engineering

Session II:

Quantum Reflection of BECs from semiconductor surfaces

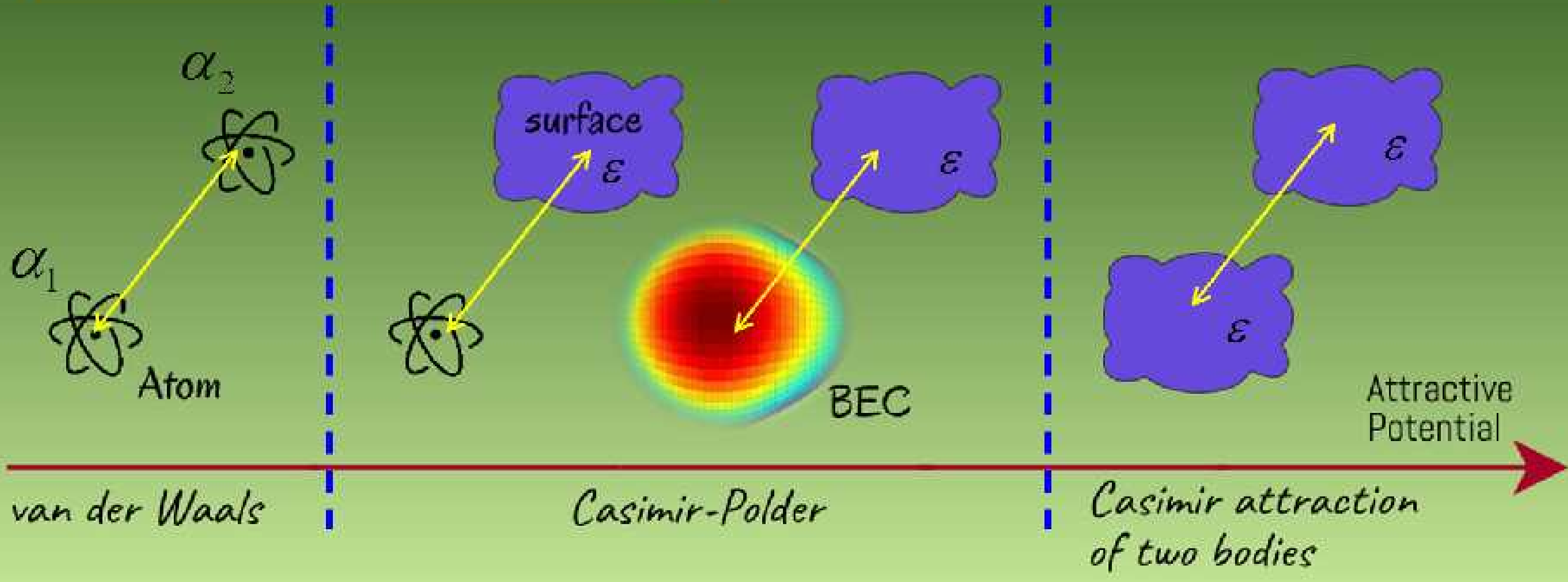
Casimir-Polder interaction

(Atom-Surface Interaction)



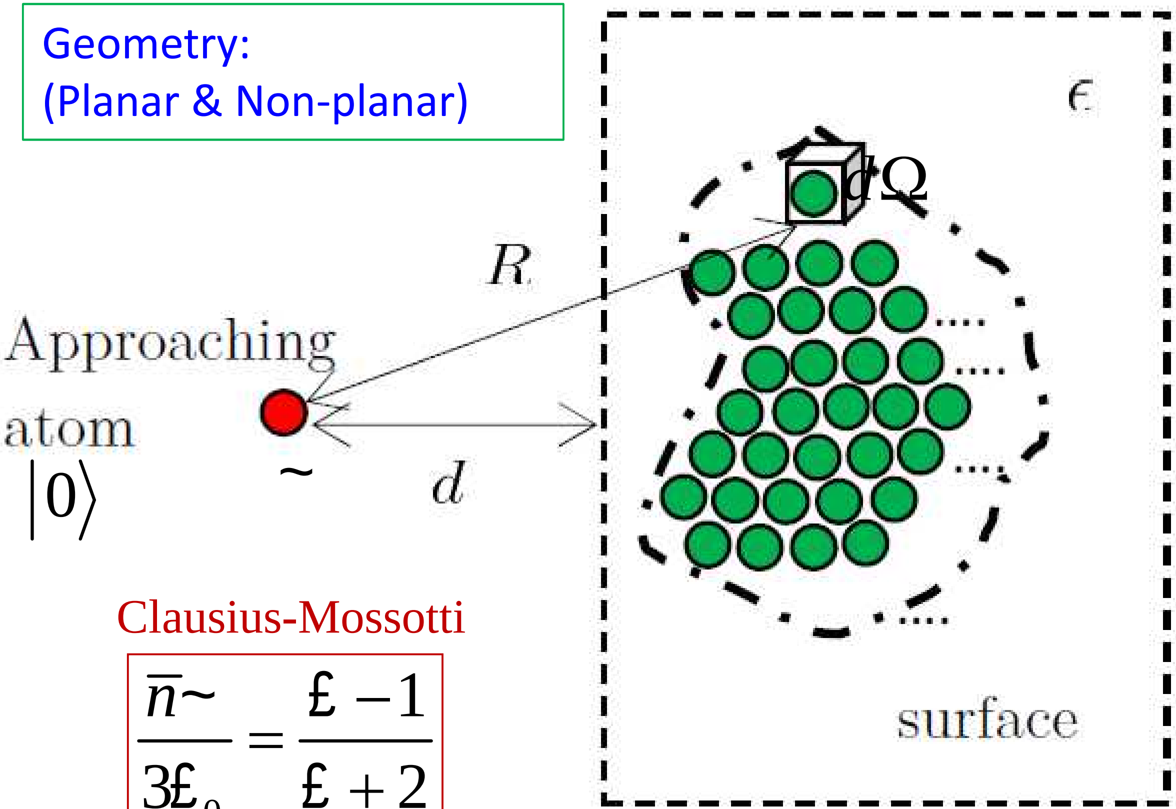
Summary of van der Waals (vdW) Casimir effects between atoms and solid bodies.

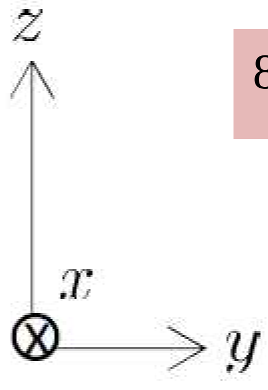
Zero-point energy (ZPE): at 0 Kelvin, $|0\rangle$



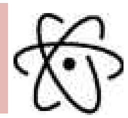
Casimir-Polder (CP) Interaction

Geometry:
(Planar & Non-planar)





^{87}Rb



Clausius-Mossotti

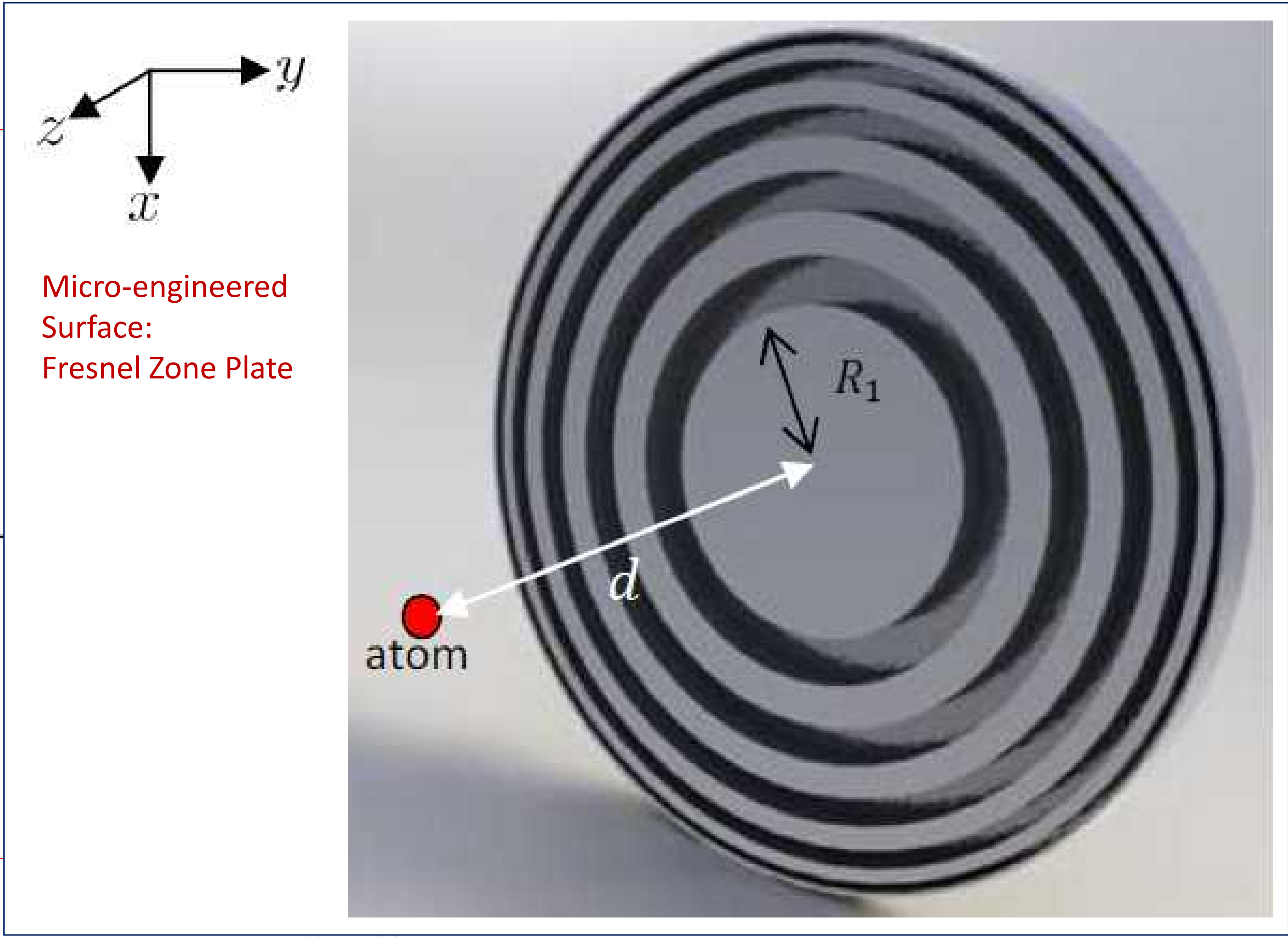
$$\frac{\bar{n}\tilde{\alpha}}{3\varepsilon_0} = \frac{\varepsilon - 1}{\varepsilon + 2}$$

Pairwise-Summation method (PWS) :

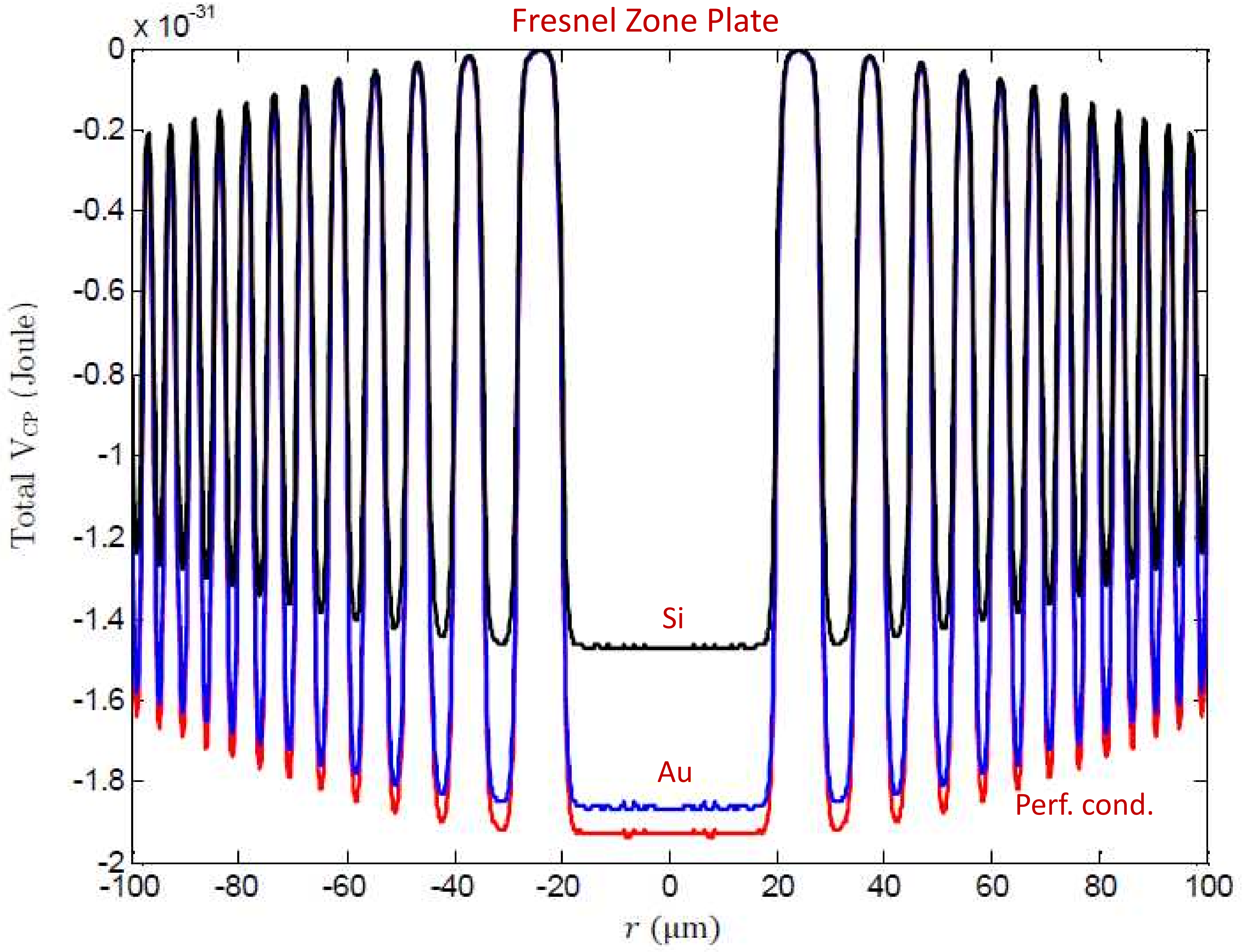
$$V_{\mathrm{CP}} = -C_m \, \bar{n} \int_{\Omega} \frac{d\Omega}{R^m}$$

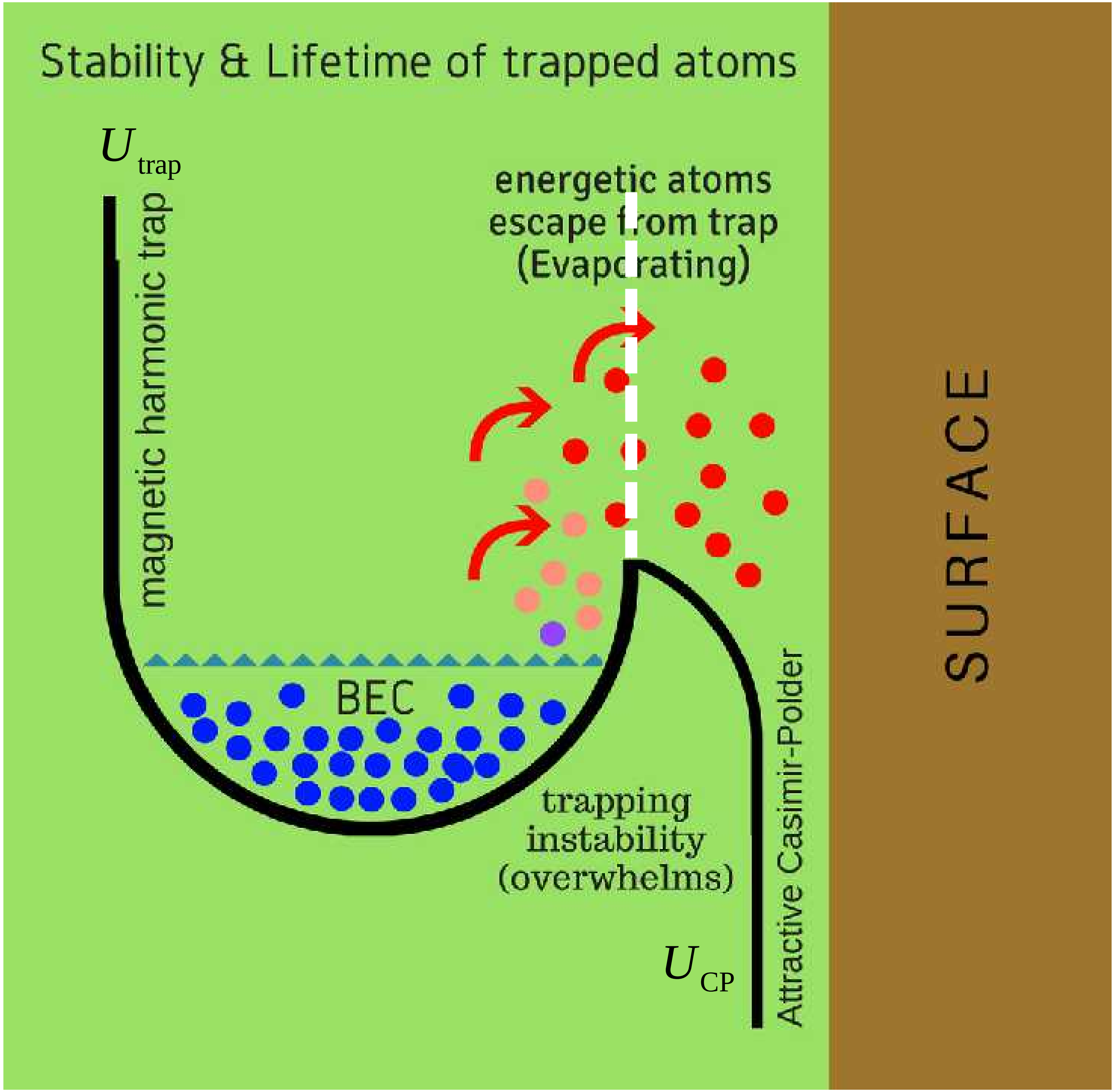
van der Waals (vdW) atom-atom interaction, where m = 6 (non-retarded, NRt) and m = 7 (retarded, Rt) regimes (interaction strength).
Integration over volume of surface Ω (GEOMETRY).
$\bar{n}$ is volume density of atoms in the surface.

$$V_{\mathrm{CP;\ total}} = \left(V_{\mathrm{NRt}}^{-1} + V_{\mathrm{Rt}}^{-1}\right)^{-1}$$



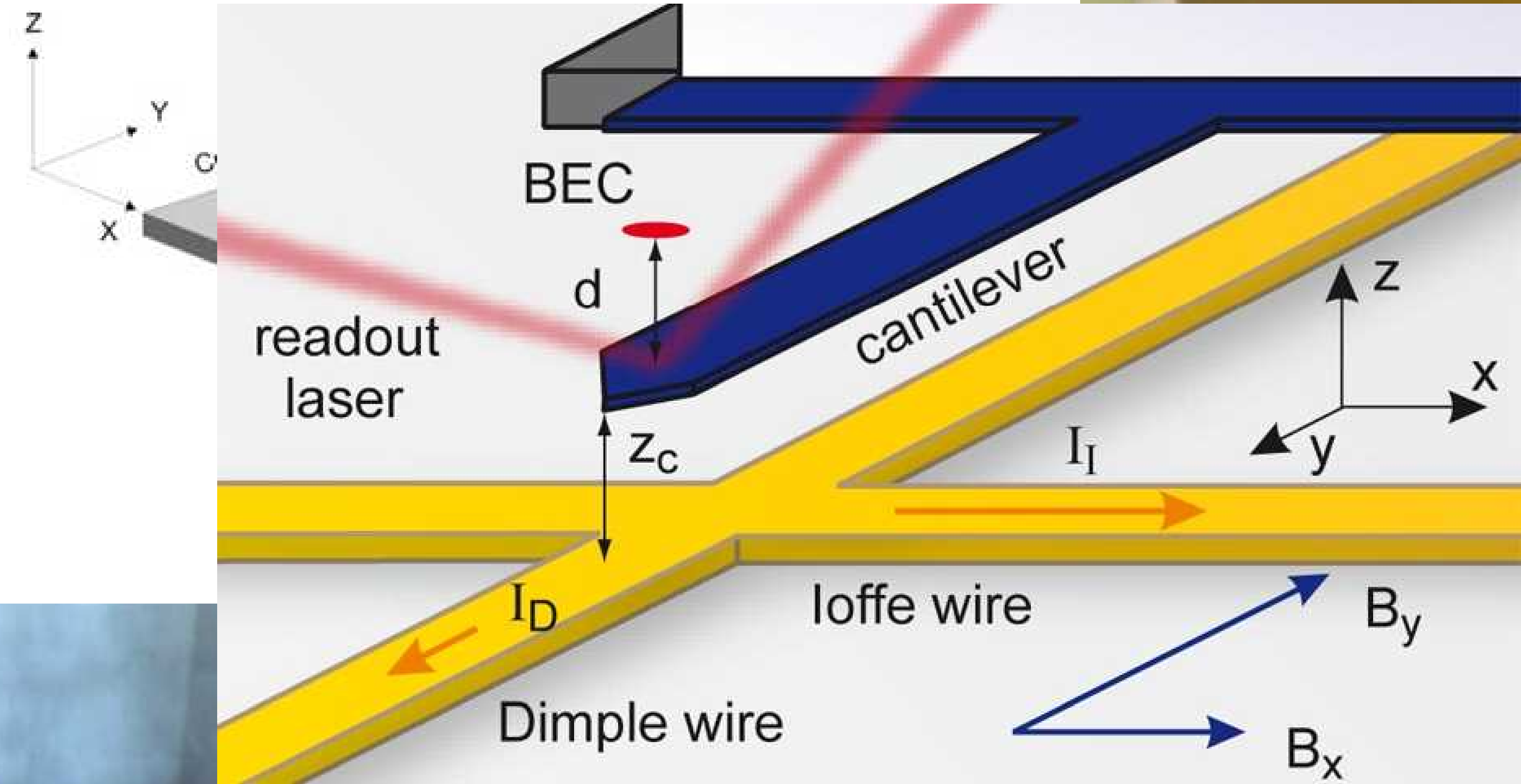
y
z
x
Micro-engineered
Surface:
Fresnel Zone Plate
R1
d
atom


Fresnel Zone Plate
x 10^-31
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
-1.8
-2
Total V_CP (Joule)
-100
-80
-60
-40
-20
0
20
40
60
80
100
r (μm)
Si
Au
Perf. cond.



Stability & Lifetime of trapped atoms
U_{trap}
magnetic harmonic trap
energetic atoms
escape from trap
(Evaporating)
BEC
trapping
instability
(overwhelms)
Attractive Casimir-Polder
U_{CP}
SURFACE

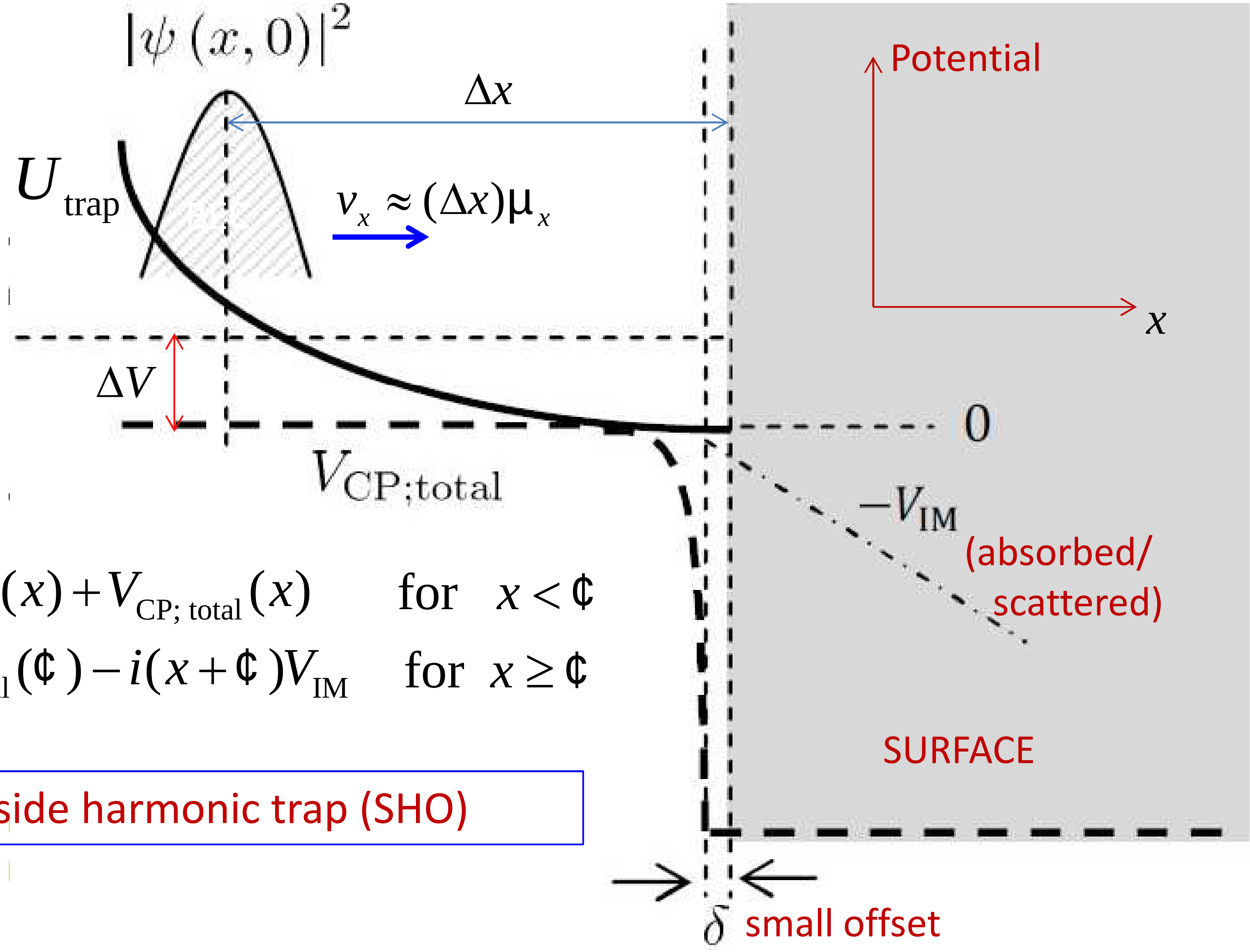
BEC & Casimir-Polder



M. Keil et al., J. Mod. Opt., 63, 1840 (2016)

Quantum Reflection of BECs

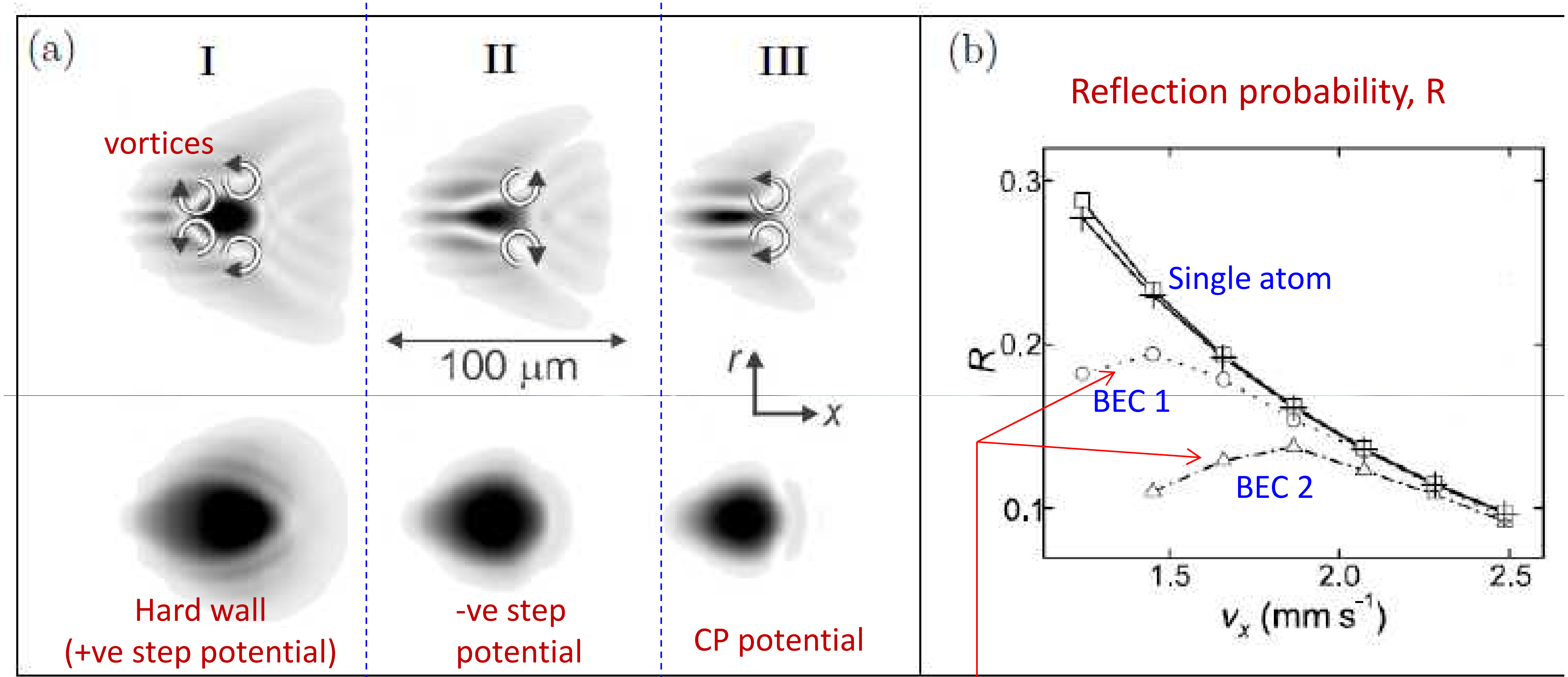
1D model



$$V_{\text{total}} = \begin{cases} U_{\text{trap}}(x) + V_{\text{CP; total}}(x) & \text{for } x < \text{¢} \\ U_{\text{CP; total}}(\text{¢}) - i(x + \text{¢})V_{\text{IM}} & \text{for } x \geq \text{¢} \end{cases}$$

BEC oscillate inside harmonic trap (SHO)

Quantum Reflection of BECs



After reflection: BEC from flat Si surface

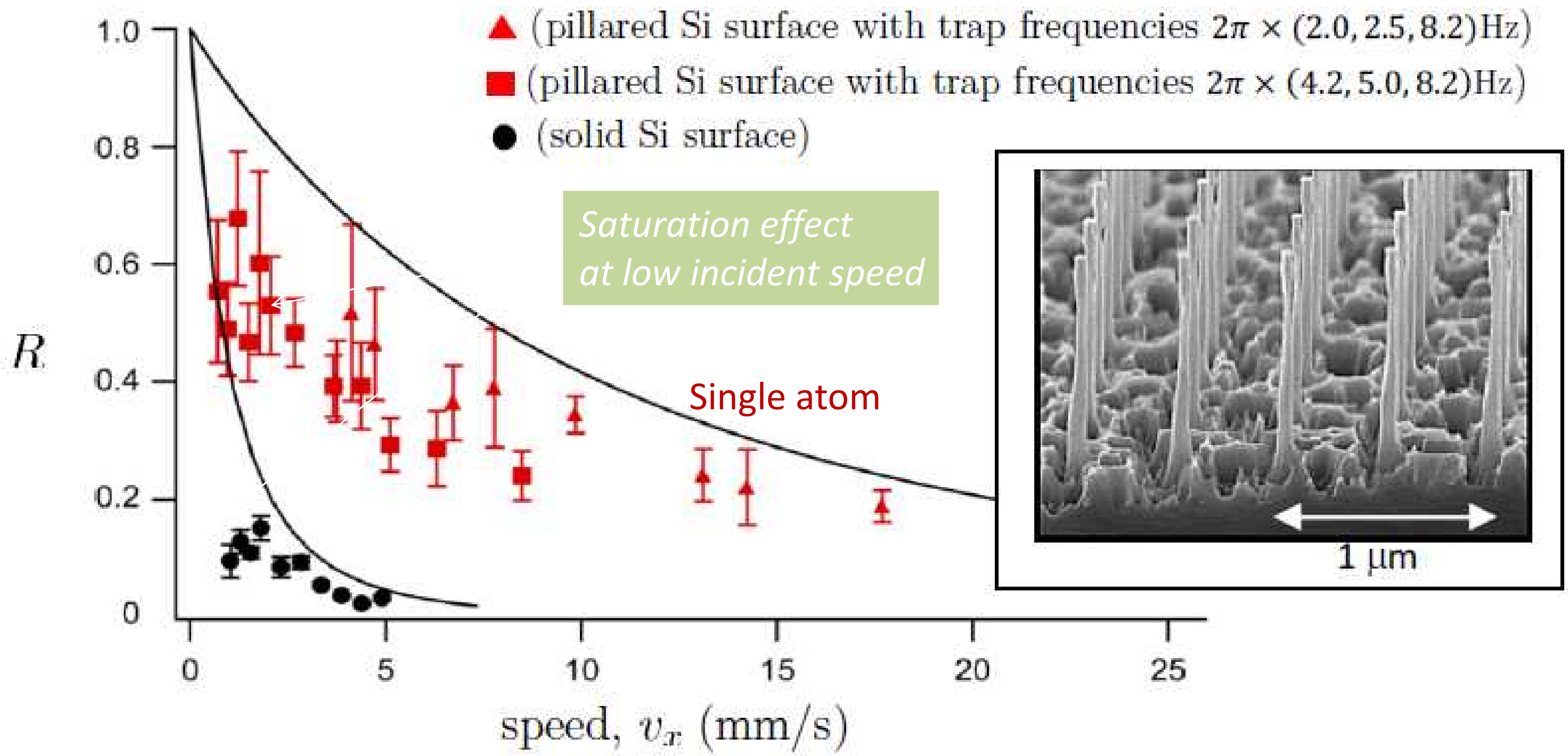
BEC 1: N = 300,000 atoms
BEC 2: N = 1,000,000 atoms

This effect in contrast with the theory of QR for a single atom (non-interacting BEC), as $v_x \rightarrow 0$, will enhance R ($R \rightarrow 1$).

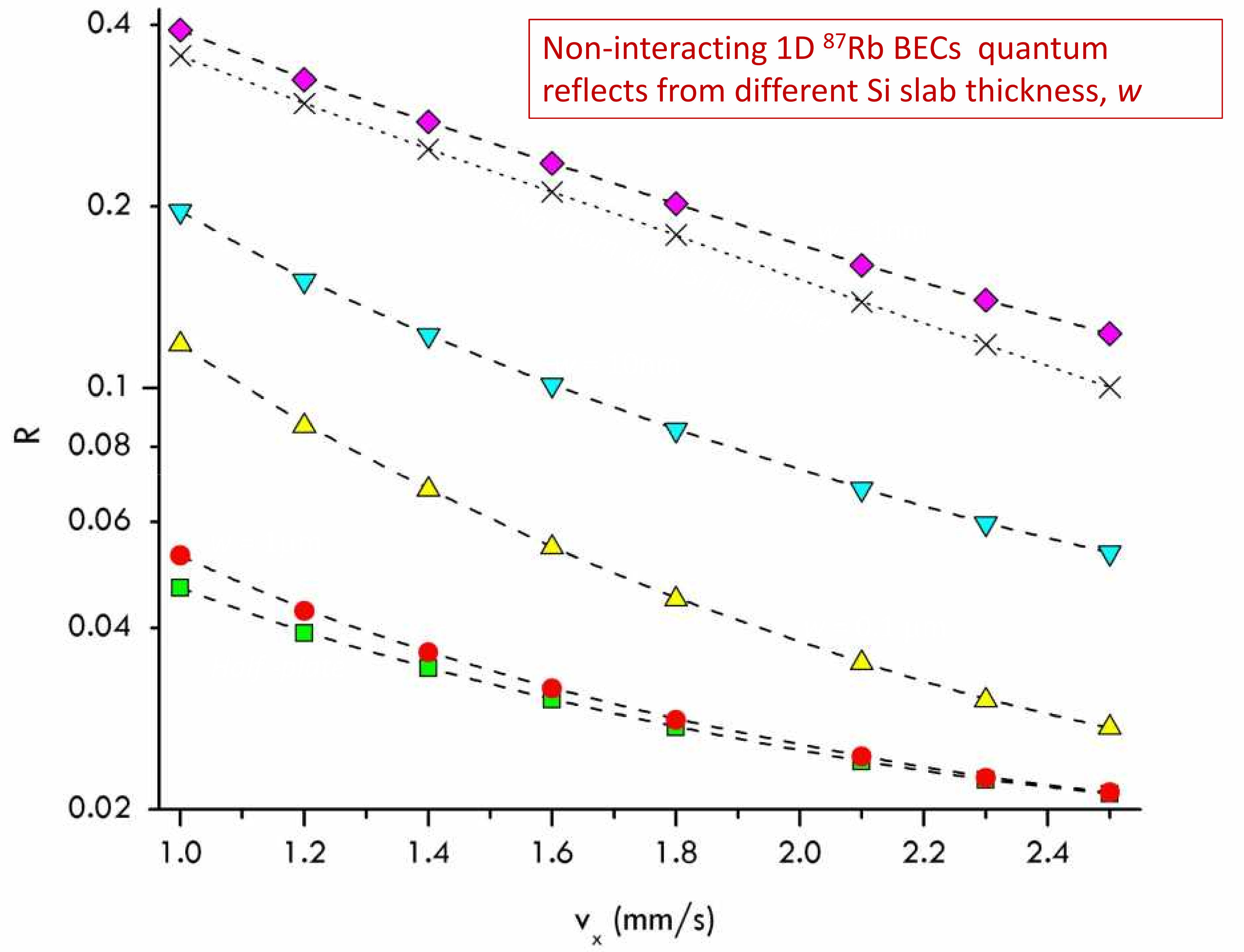
Saturation effect

Quantum Reflection of BECs

(Experimental works: MIT group)

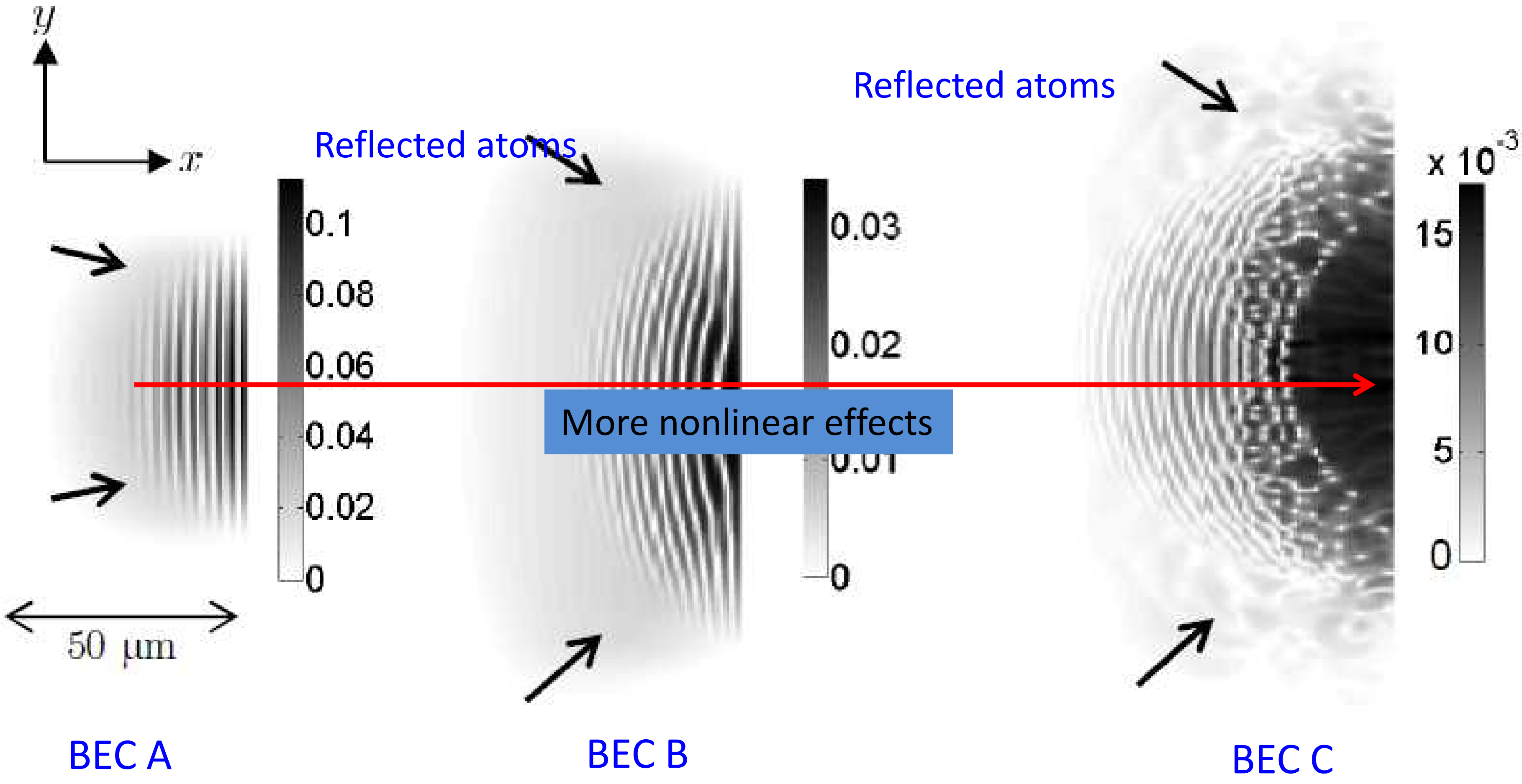


Refs: T.A Pasquini et al., PRL, 93, 223201 (2004)
T.A. Pasquini et al., PRL, 97, 093201 (2006)

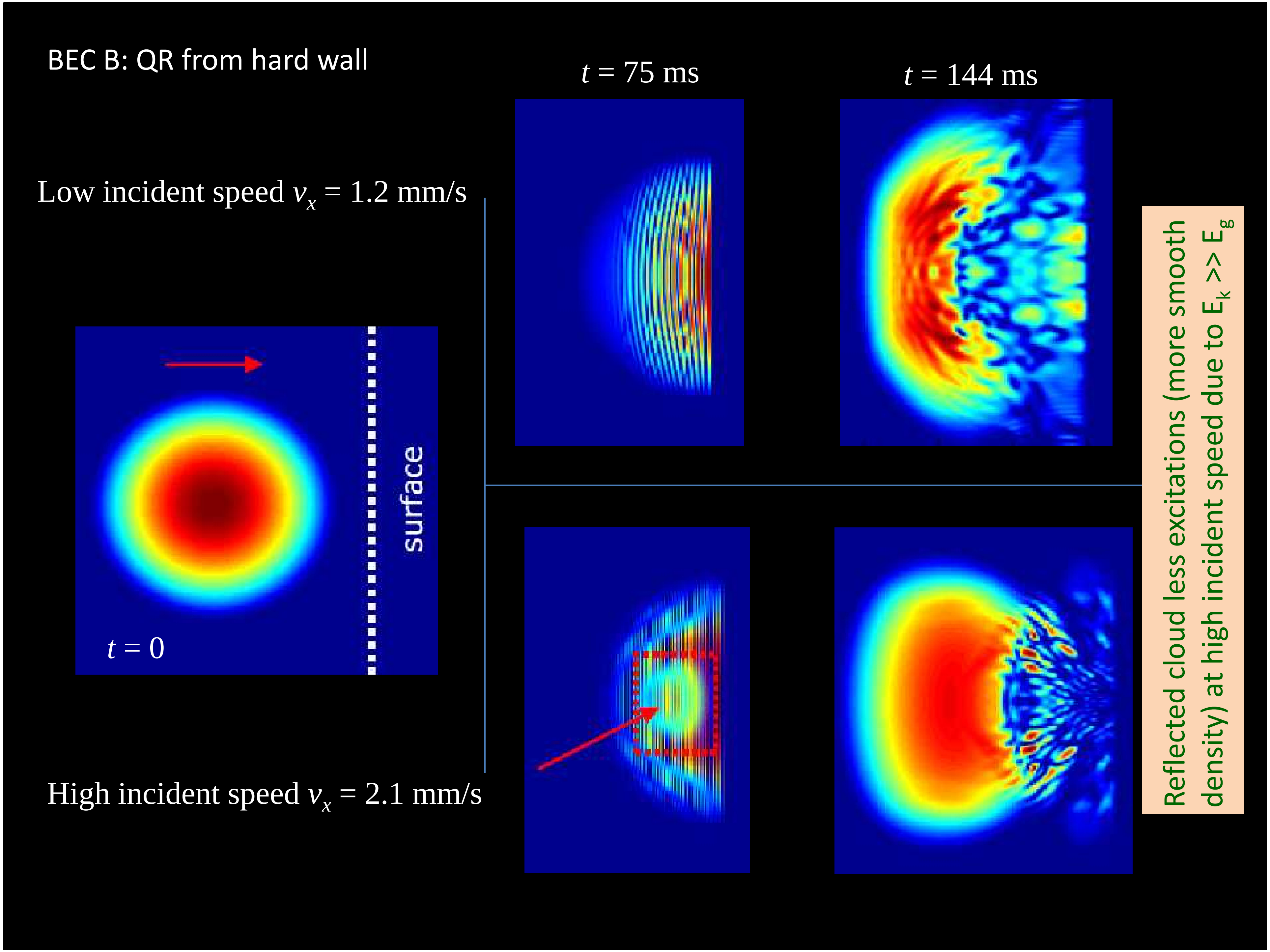


Non-interacting 1D ^{87}Rb BECs quantum reflects from different Si slab thickness, *w*
0.4
0.2
0.1
0.08
0.06
0.04
0.02
R
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
v_x (mm/s)

Quantum Reflection of BECs



QR of the BECs from a hard wall Si surface with v_x = 1.2 mm/s at t = 90ms (black = high density)

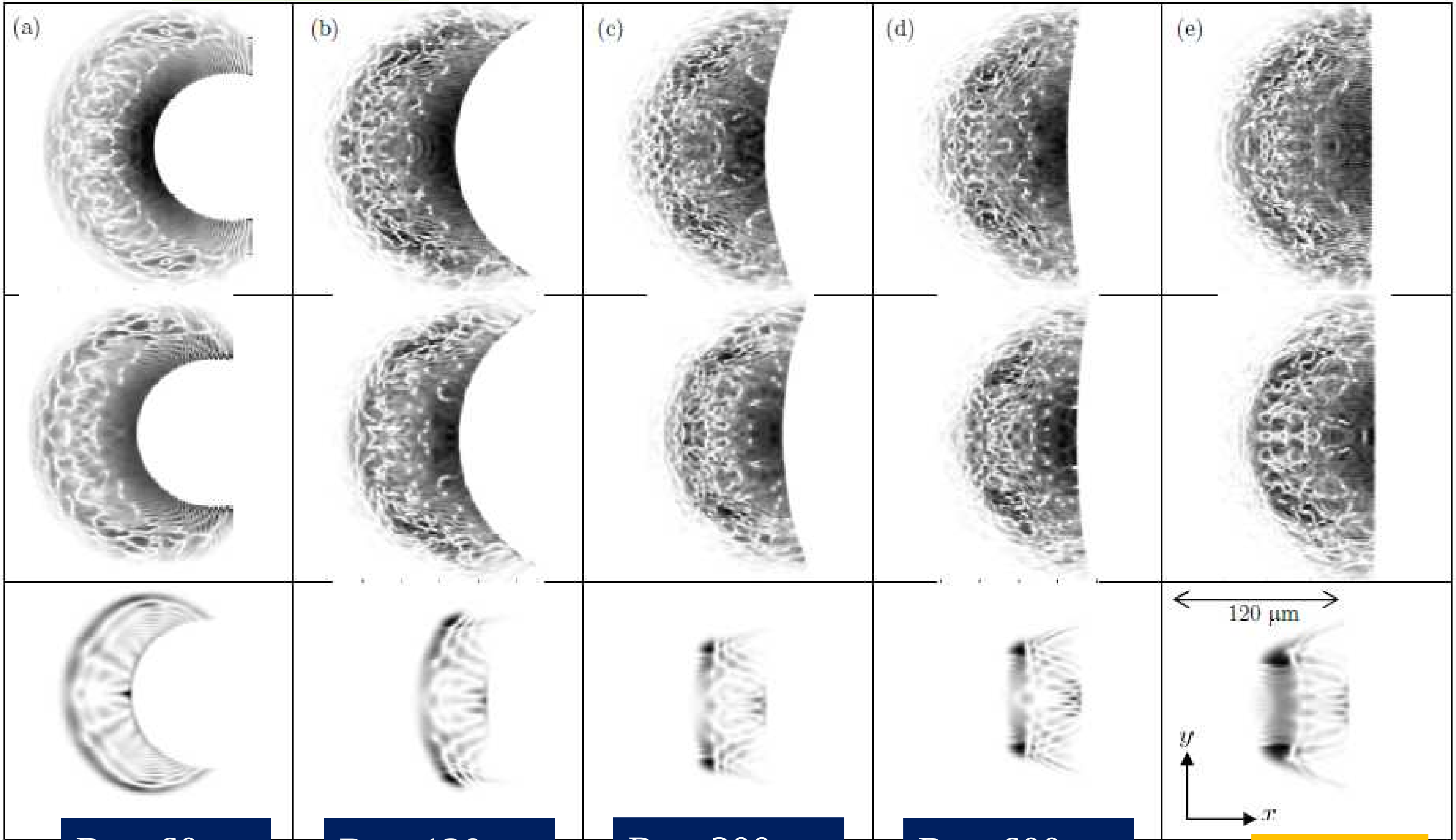


BEC B: QR from hard wall
t = 75 ms
t = 144 ms
Low incident speed v_x = 1.2 mm/s
surface
t = 0
High incident speed v_x = 2.1 mm/s
Reflected cloud less excitations (more smooth density) at high incident speed due to $E_k >> E_g$

Quantum Reflection of BECs
from cylindrical (curved) surface

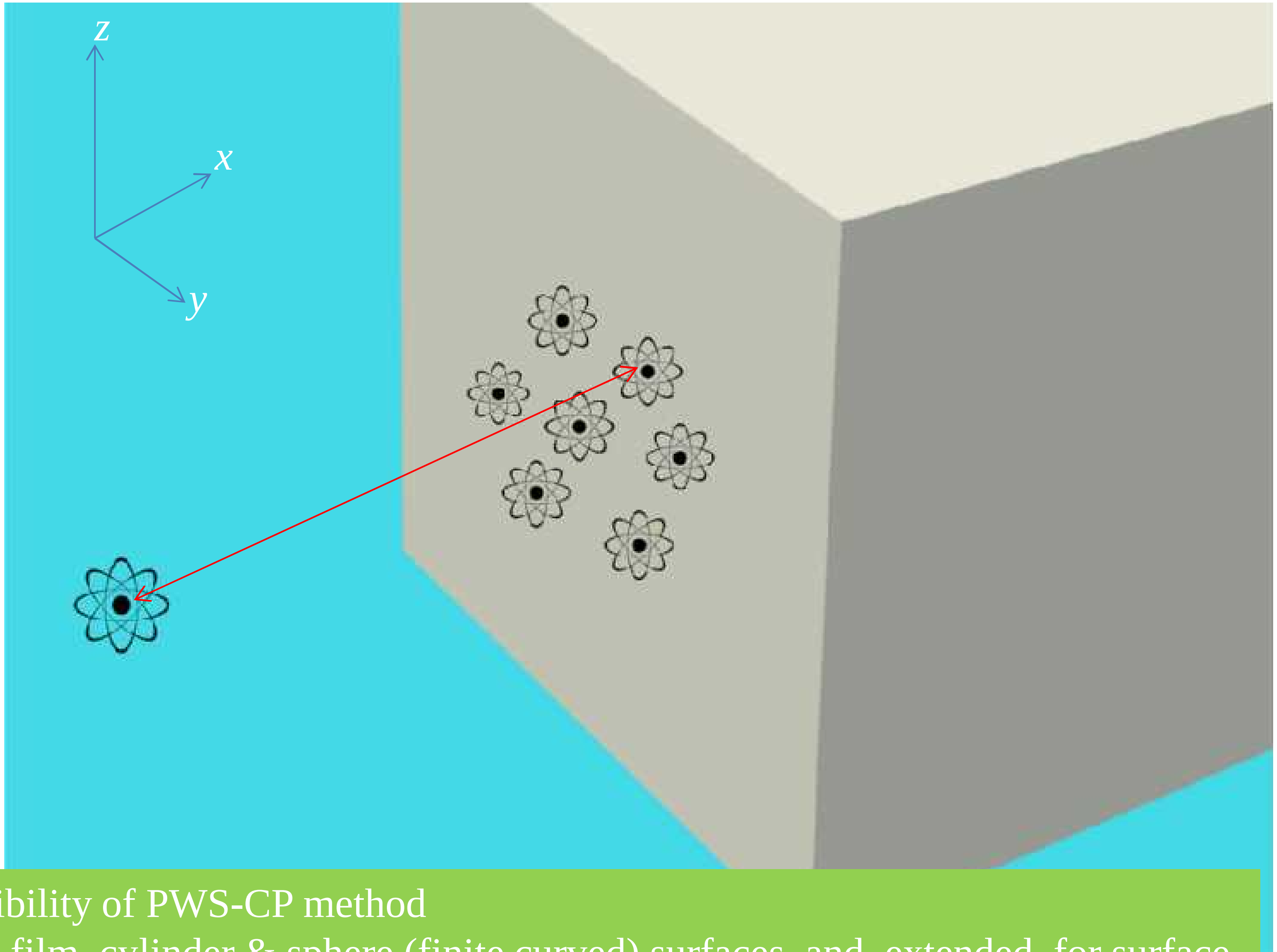
BEC C

$v_x = 1.2 mm/s$



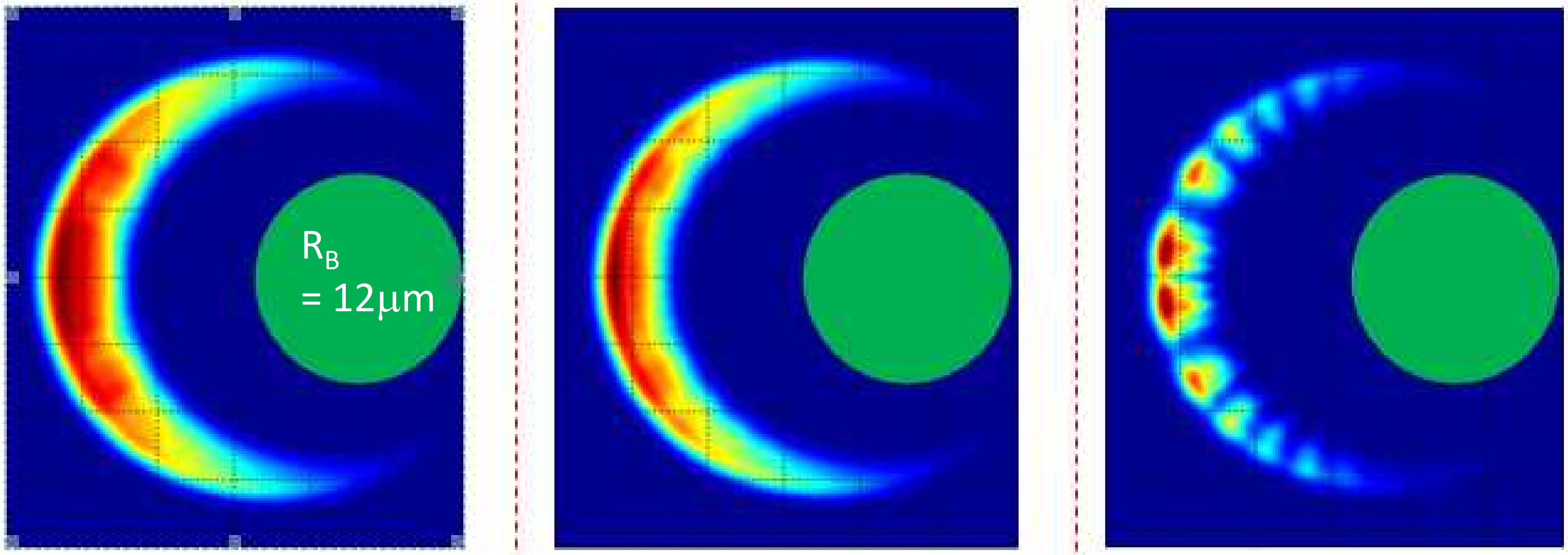
$R_c = 60 \mu m$ | $R_c = 120 \mu m$ | $R_c = 300 \mu m$ | $R_c = 600 \mu m$ | Half plate

Current & Future works



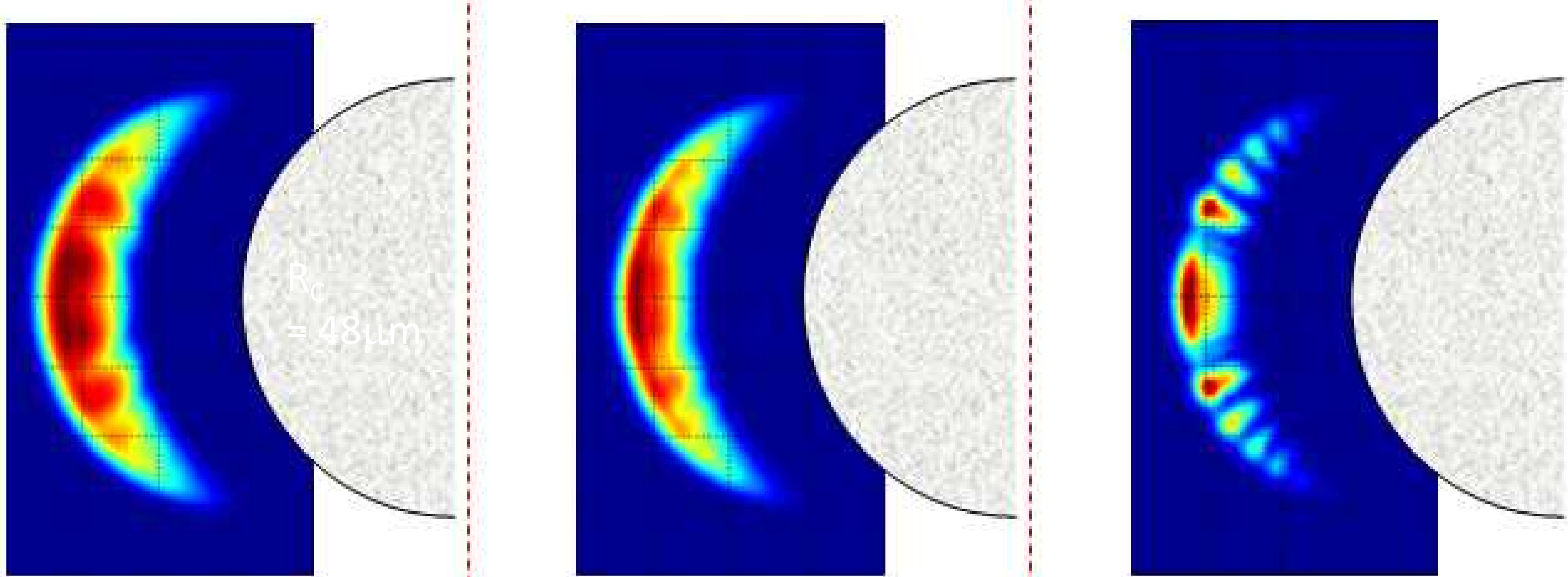
Flexibility of PWS-CP method
from film, cylinder & sphere (finite curved) surfaces, and, extended for surface temperature effect (attractive to repulsive) – MSc (on-going)

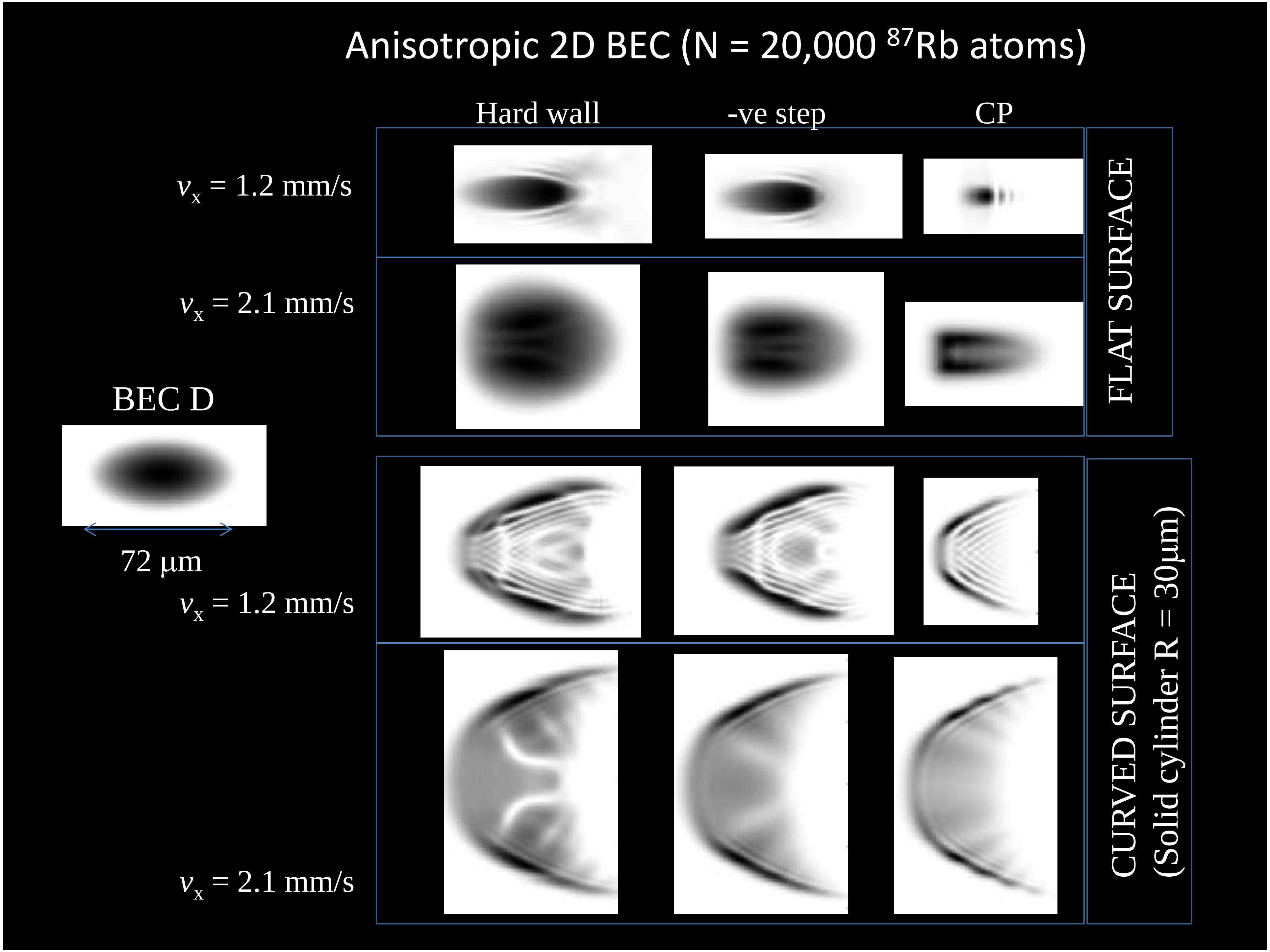
Cylinder B



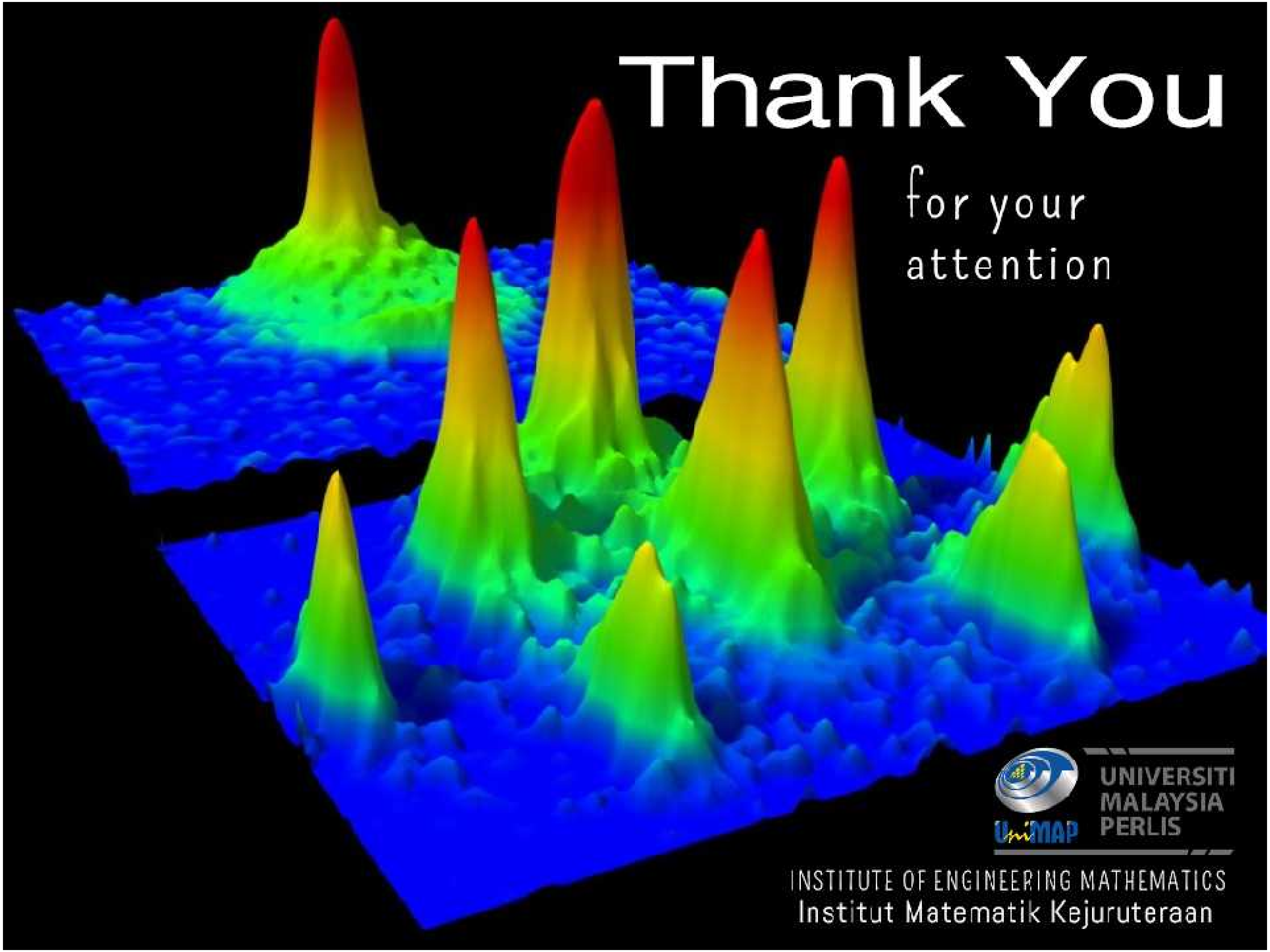
Cylinder C

BEC A at v_x = 1.2 mm/s





Anisotropic 2D BEC (N = 20,000 ^{87}Rb atoms)
Hard wall
-ve step
CP
v_x = 1.2 mm/s
v_x = 2.1 mm/s
FLAT SURFACE
BEC D
72 μm
v_x = 1.2 mm/s
v_x = 2.1 mm/s
CURVED SURFACE
(Solid cylinder R = 30μm)


Thank You
for your attention
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UniMAP
INSTITUTE OF ENGINEERING MATHEMATICS
Institut Matematik Kejuruteraan

The Gross-Pitaevskii Equation

a nonlinear Schrodinger Equation

and

Bose-Einstein Condensates (BEC)

Low Temperature, atom & waves

- At very low temperature the de Broglie wavelengths of the atoms are very large compared to the range of the interatomic potential.
- This, together with the fact that the density and energy of the atoms are so low that they rarely approach each other very closely, means that atom–atom interactions are effectively *weak* and dominated by (elastic) *s*-wave scattering.

- The *s*-wave scattering length ”*a*” the sign of which depends sensitively on the precise details of the interatomic potential.

* $a > 0$ for repulsive interactions.

* $a < 0$ for attractive interactions.

- In the Bose-Einstein Condensation, the majority of the atoms condense into the same single particle quantum state and lose their individuality (*identity crisis*).

Low Temperature, atom & waves

- Since any given atom is not aware of the individual behaviour of its neighbouring atoms in the condensate, the interaction of the cloud with any single atom can be approximated by the cloud's mean field, and the whole ensemble can be described by the same single particle wavefunction.

- In |g>, each of the *N* particles occupies a definite single-particle state, so that its motion is independent of the presence of the other particles.

- Hence, a natural approach is to assume that each particle moves in a single-particle potential that comes from its average interaction with all the other particles.

- This is the definition of the self-consistent mean-field approximation.

Mean-field theory (concept)

- Decompose wave function into two parts:

1) The condensate wave function, which is the expectation value of wave function.
2) The non-condensate wave function, which describes quantum and thermal fluctuations around this mean value but can be ignored due to ultra-cold temperature.

The Mean-Field Approximation

- The many-body Hamiltonian describing N interacting bosons confined by an external potential V_{trap} is given, in second quantization, by

$$\hat{H} = \int d^3\mathbf{r}\hat{\Psi}^\dagger(\mathbf{r},t)\left[-\frac{\hbar^2\nabla_\mathbf{r}^2}{2m} + V_{trap}(\mathbf{r},t)\right]\hat{\Psi}(\mathbf{r},t)$$

$$+\frac{1}{2}\int\int d^3\mathbf{r}d^3\mathbf{r}'\hat{\Psi}^\dagger(\mathbf{r},t)\hat{\Psi}^\dagger(\mathbf{r}',t)V(\mathbf{r}-\mathbf{r}')\hat{\Psi}(\mathbf{r}',t)\hat{\Psi}(\mathbf{r},t)$$

boson field operators that create and annihilate particle at the position *r*, respectively.

V(r-r') is the two body interatomic potential (interaction potential) related to the s-wave scattering length *a*

The Mean-Field Approximation

The boson field operators $\hat{\Psi}(\mathbf{r},t)$ satisfy the following commutation relations:

$$\left[\hat{\Psi}(\mathbf{r},t),\hat{\Psi}(\mathbf{r}',t)\right]=\left[\hat{\Psi}^{\dagger}(\mathbf{r},t),\hat{\Psi}^{\dagger}(\mathbf{r}',t)\right]=0$$

$$\left[\hat{\Psi}(\mathbf{r},t),\hat{\Psi}^{\dagger}(\mathbf{r}',t)\right]=\delta(\mathbf{r}-\mathbf{r}')$$

- The field operators can in general be written as a sum over all participating single-particle wave functions and the corresponding boson creation and annihilation operators.

$$\hat{\Psi}(\vec{r},t)=\sum_i \Psi_i(\vec{r},t)\hat{a}_i$$

Gross-Piteavskii Equation (GPE)

$$i\hbar \frac{\partial}{\partial t} \cdot (\tilde{r}, t) = -\frac{\hbar^2}{2m} \nabla^2 \cdot (\tilde{r}, t) + V_{\text{ext}} \cdot (\tilde{r}, t) + g \left| \cdot (\tilde{r}, t) \right|^2 \cdot (\tilde{r}, t)$$

- Schrodinger equation with nonlinear term cause by atom-atom interaction. The interaction constant given by

$$g = \frac{4\text{®}\,\hbar^2 a N}{m}$$

- The order parameter, $\cdot$, is normalized to the total number of atom, *N*, as follows:

$$\int \left| \cdot (\tilde{r}, t) \right|^2 d\tilde{r} = N$$

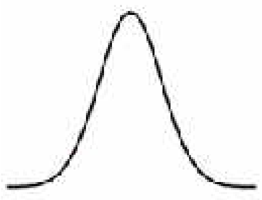
- The external trap potential V_{ext} often represented as a harmonic 3D:

$$V_{\text{ext}}(\tilde{r}) = \frac{m}{2}\left(\mu_x^2 x^2 + \mu_y^2 y^2 + \mu_z^2 z^2 \right)$$

Gross-Piteavskii Equation (GPE)

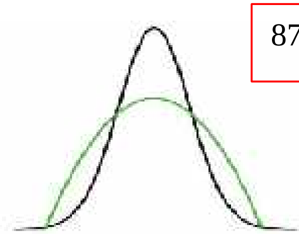
- BEC in dependence of mean field: $g = \frac{4\pi \hbar^2 a N}{m}$
- At very low temperature, one parameter sufficient to describe interaction is scattering length, a, where:

(i) Ideal gas, $a = 0$ (linear case)



Non-interacting BEC
(Gaussian)

(ii) Repulsive, $a > 0$ (nonlinear case)



^{87}Rb, ^{23}Na

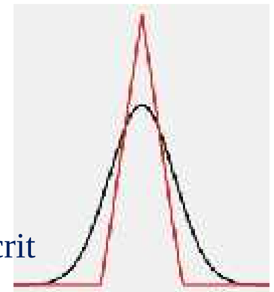
Dark soliton

Interacting BEC
(Parabola)

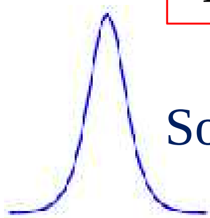
(iii) Attractive, $a < 0$ (nonlinear case)

(ii) & (iii) $a \neq 0$
(non-spreading wave-packet (solition) are possible

3D
Collapse for $N > N_{crit}$



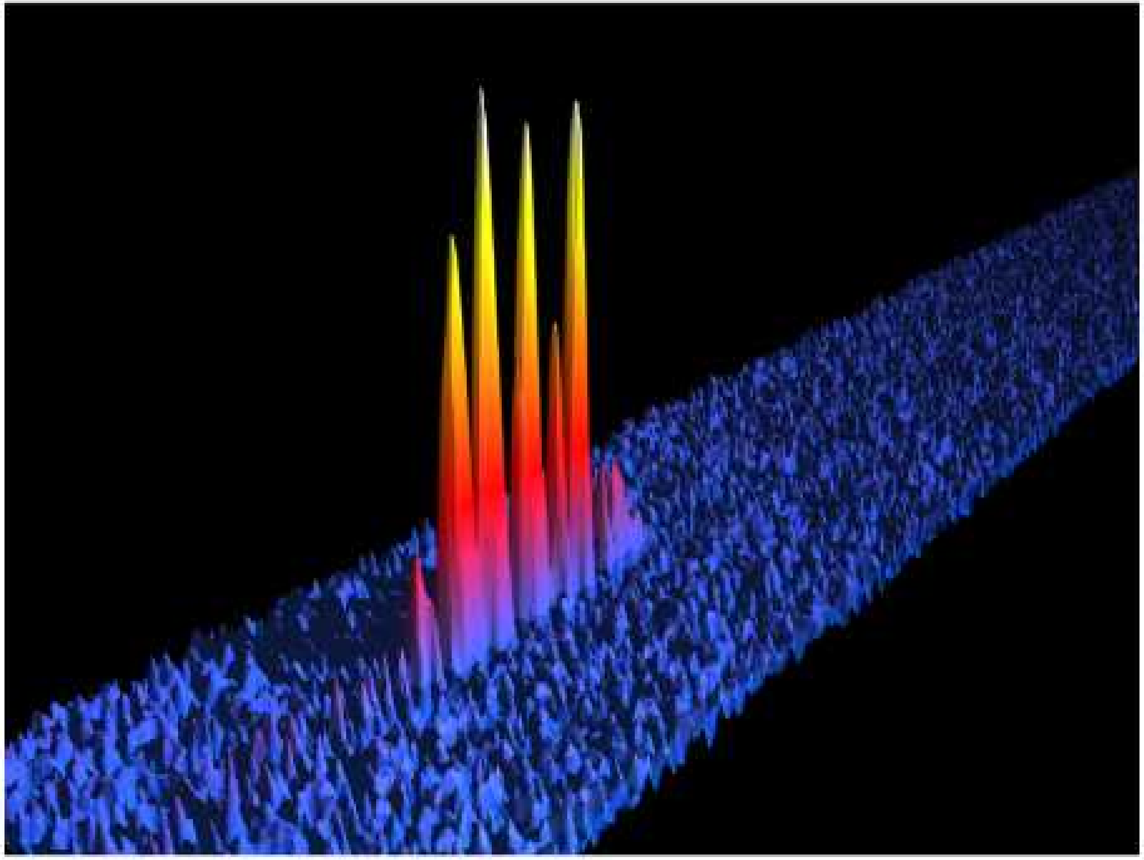
^{7}Li



1D
Solition

Bright soliton

The famous experimental picture of the density distribution in a ***chain of 7 solitons*** in **Li-7** (produced by the group of *R. Hulet*):



Gross-Piteavskii Equation (GPE)

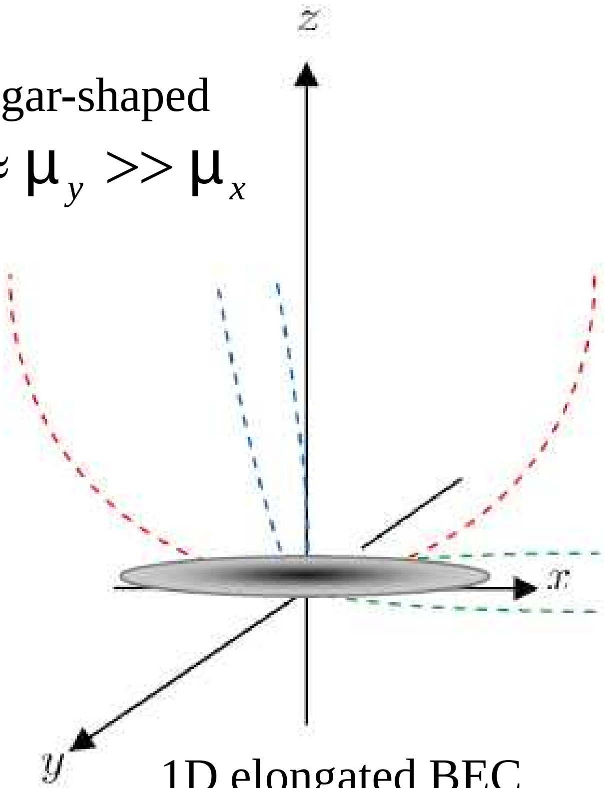
Matter-Wave Soliton BEC:

- Better understanding of the stability, as well as the static and dynamical properties of matter-wave dark soliton.
- In experiments (**not directly related to dark solitons**) – reported observation of these structure (i.e, manipulation of BEC near dielectric surfaces – Quantum Reflection of BEC)

1D and 2D Condensates (harmonic trap)

1D cigar-shaped

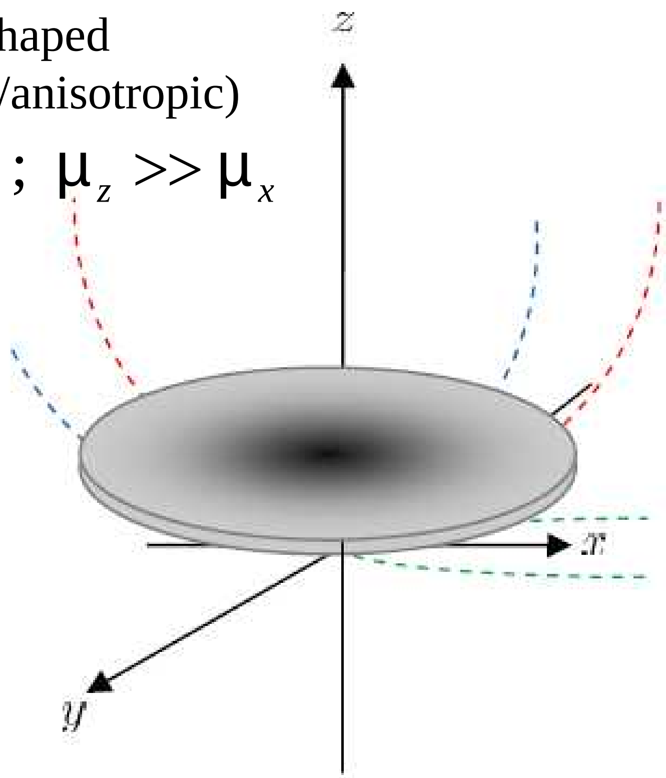
$\mu_z \approx \mu_y >> \mu_x$



1D elongated BEC

2D disc-shaped
(isotropic/anisotropic)

$\mu_x \approx \mu_y \;\; ; \;\; \mu_z >> \mu_x$



$$U_{\text{trap}}(x, y, z) = \frac{1}{2} m \left(\mu_x^2 x^2 + \mu_y^2 y^2 + \mu_z^2 z^2 \right)$$

~ 2π x 3.3Hz

Using imaginary-time method to simulate GPE – ground state BEC
(Number of atoms, N, and types of atom (mass))

Ground state BEC

$$i\hbar \frac{\partial}{\partial t} \cdot (\tilde{r}, t) = -\frac{\hbar^2}{2m} \nabla^2 \cdot (\tilde{r}, t) + V_{\text{ext}} \cdot (\tilde{r}, t) + g \left| \cdot (\tilde{r}, t) \right|^2 \cdot (\tilde{r}, t)$$

$$g_j = \begin{cases} 2aN\hbar\mu_\perp \\ \dfrac{\sqrt{8\circledR}\,\hbar^2 aN}{m}\sqrt{\dfrac{m\mu_z}{\hbar}} \\ \dfrac{4\circledR\,\hbar^2 aN}{m} \end{cases} \quad ; \quad V_j = \begin{cases} \dfrac{1}{2} m\mu_x^2 x^2 \\ \dfrac{1}{2} m\left(\mu_x^2 x^2 + \mu_y^2 y^2\right) \\ \dfrac{1}{2} m\left(\mu_x^2 x^2 + \mu_y^2 y^2 + \mu_z^2 z^2\right) \end{cases}$$

$j = 1, 2, 3$

Dimensionless GPE, using scaling parameters:

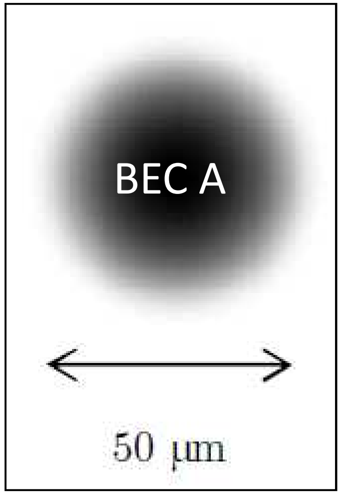
$\mu_x = 2\circledR \times 3.3 \text{ Hz}$

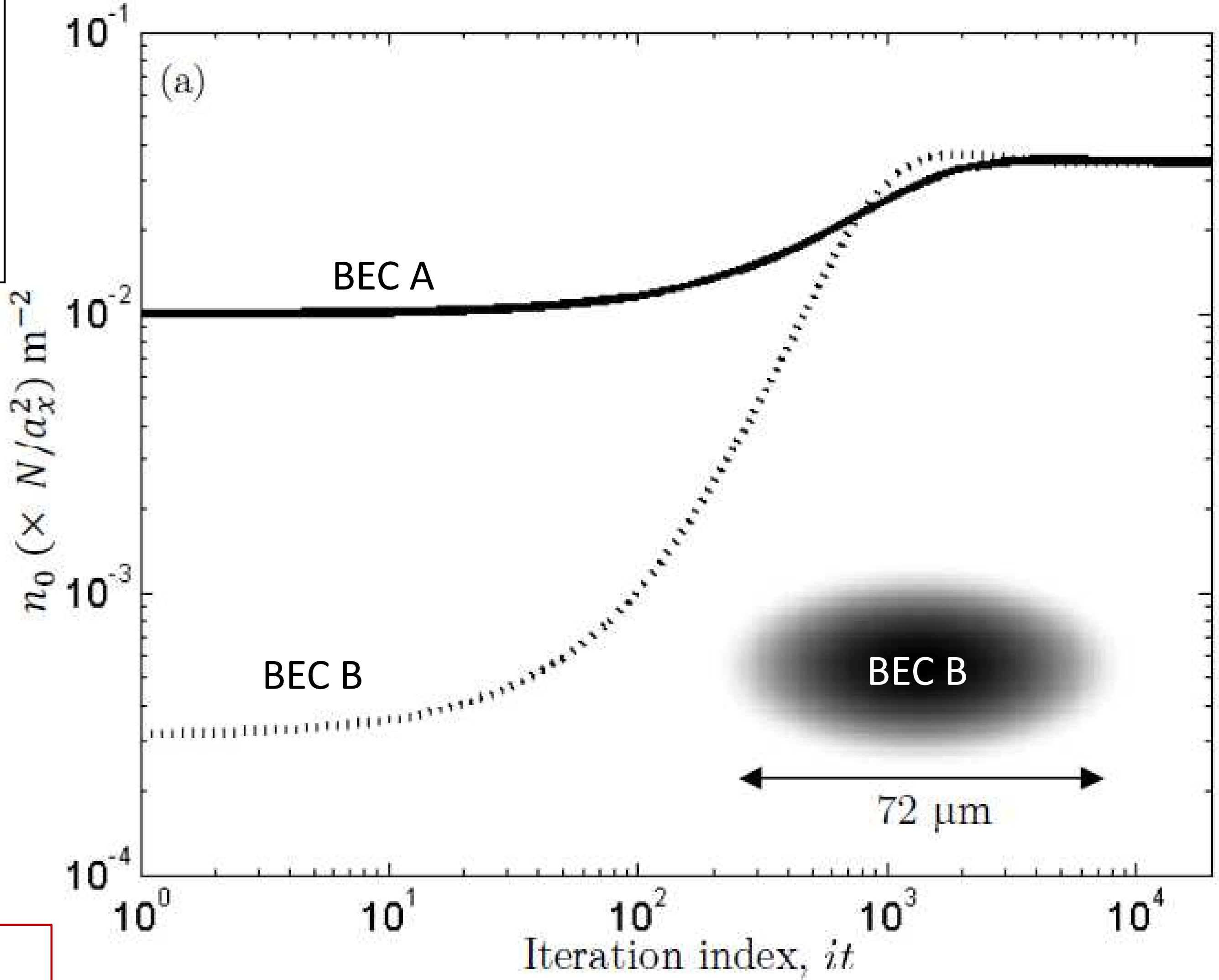
$a_x = \sqrt{\dfrac{\hbar}{m\mu_x}}$

$t = \dfrac{2}{\mu_x}$

Ground State: The Imaginary Time Method ($\Delta t \rightarrow - i\Delta t$)

Ground state BEC





Density profile:
Black = high density

Manipulating BECs

$$i\hbar \frac{\partial}{\partial t} \cdot \ (\tilde{r},t) = -\frac{\hbar^2}{2m}\nabla^2 \cdot \ (\tilde{r},t) + V_{\mathrm{ext}} \cdot \ (\tilde{r},t) + g\left|\cdot \ (\tilde{r},t)\right|^2 \cdot \ (\tilde{r},t)$$

Harmonic Trap + Potential barrier *

* Atom-surface interaction (Casimir-Polder theory)
* Gaussian barrier

Off harmonic trap (expansion of BEC)

- Species of atom
- Modified mean-field energy
- Many-body interaction with surface

Soliton in BEC

- Dark soliton in BEC (theoretical) started as early as 1971 (T. Tsuzuki , *J. Low Temp. Phys.* 4, 441 (1971)) and a new era for dark solitons started shortly after the realization of atomic BECs in 1995.
- See: D. J. Frantzeskakis "*Dark solitons in atomic Bose-Einstein condensates: from theory to experiment*s" arXiv:1004.4071v1

- One of the example of manipulating BEC to create solition: Quantum Reflection of BEC from dielectric surfaces (semiconductor surfaces)