MATEMATIKA FUZZY DAN APLIKASINYA DALAM KEHIDUPAN NYATA

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What is Fuzzy Mathematics?

- Fuzzy mathematics forms a branch of mathematics related to fuzzy set theory and fuzzy logic.
  - Fuzzy sets (uncertain sets) are somewhat like sets whose elements have degrees of membership (Lotfi A. Zadeh, 1965).
  - Fuzzy logic is a form of many-valued logic in which the truth values of variables may be any real number between 0 and 1 both inclusive.

- Fuzziness can be found in many areas of daily life, such as in engineering, medicine, meteorology, manufacturing, economy, and others. It is particularly frequent, however, in all areas in which human judgment, evaluation, and decisions are important.
Fuzzy subgroupoids and fuzzy subgroups were introduced in 1971 by A. Rosenfeld.

Fuzzy fields and fuzzy Galois theory are published in a 1998 paper. (Mordeson & Malik)

Fuzzy topology was introduced by C.L. Chang in 1968

Fuzzy geometry were introduced by Tim Poston in 1971, A. Rosenfeld in 1974, by J.J. Buckley and E. Eslami in 1997 and by D. Ghosh and D. Chakraborty in 2012-14.

Basic types of fuzzy relations were introduced by Zadeh in 1971.

Fuzzy graphs have been studied by A. Kaufman, A. Rosenfel, and by R.T. Yeh and S.Y. Bang.
Fuzzy sets

- In mathematics fuzzy sets have triggered new research topics in connection with category theory, topology, algebra, analysis.
- Fuzzy sets are also part of a recent trend in the study of generalized measures and integrals, and are combined with statistical methods.
- In decision and organization sciences, fuzzy sets has had a great impact in preference modeling and multicriteria evaluation, and has helped bringing optimization techniques closer to the users needs.
- Applications can be found in many areas such as management, production research, and finance. Moreover concepts and methods of fuzzy set theory have attracted scientists in many other disciplines pertaining to human-oriented studies such as cognitive psychology and some aspects of social sciences.
The concept of a set and set theory are powerful concepts in mathematics. However, the principal notion underlying set theory, that an element can (exclusively) either belong to set or not belong to a set, makes it well nigh impossible to represent much of human discourse.

How is one to represent notions like:
- large profit
- high pressure
- tall man
- moderate temperature
A crisp set, $A$, can be defined as a set which consists of elements with either full or no membership at all in the set. Each item in its universe is either in the set, or not.

A fuzzy set is defined as a class of objects with a continuum of grades of membership. It is characterized by a membership function or characteristic function that assigns to each member of the fuzzy set a degree of membership in the unit interval $[0,1]$.

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MATEMATIKA FUZZY DAN APLIKASINYA DALAM KEHIDUP
Crisp and Fuzzy example

X = \{\}

One can define the crisp set “circles” as:

C = \{\}

The fuzzy set “circles can be defined as:

C = \{((\text{black octagon}, 0.1), (\text{black octagon}, 0.3), (\text{black circle}, 0.5), (\text{black circle}, 0.8), (\text{white circle}, 1.0), (\text{grey circle}, 1.0)), (\text{grey circle}, 1.0))\}
**Definition (Molodtsov, 1999)**

Let $U$ be an initial universe set and let $E$ be a set of parameters. A pair $(F, E)$ is called a soft set over $U$ if and only if $F$ is a mapping of $E$ into the set of all subsets of the set $P(U)$, i.e. $F : E \to P(U)$, where $P(U)$ is the power set of $U$.

**Example**

Let $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ be a set of houses under consideration where $E = \{e_1, e_2, e_3, e_4, e_5\}$ and $A = \{e_1, e_2, e_3, e_4\}$ be a subset of parameters for selection of the house. Let $e_1$ stands for expensive houses, $e_2$ stands for wooden houses, $e_3$ stands for houses located in green surroundings, $e_4$ stands for houses located in the urban area, $e_5$ stands for the low cost houses. Let $(F, A)$ be the soft set to categorize the houses with respect to parameters given by set $A$, such that $F(e_1) = \{h_1, h_3\}$, $F(e_2) = \{h_1, h_3, h_6\}$, $F(e_3) = \{h_1, h_3, h_4, h_5\}$, $F(e_4) = \{h_1, h_2, h_3\}$. 
For computer applications it is more appropriate to represent a soft set in tabular form.
Let $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$, be a set of six houses,
$E = \{\text{expensive}=e_1; \text{beautiful}=e_2; \text{wooden}=e_3; \text{cheap}=e_4; \text{in the green surroundings}=e_5; \text{modern}=e_6; \text{in good repair}=e_7; \text{in bad repair}=e_8\}$, be a set of parameters.

Consider the soft set $(F, E)$ which describes the attractiveness of the houses, given by $(F, E) = \{\text{expensive houses} = \emptyset, \text{beautiful houses} = \{h_1, h_2, h_3, h_4, h_5, h_6\}, \text{wooden houses} = \{h_1, h_2, h_6\}, \text{modern houses} = \{h_1, h_2, h_6\}, \text{in bad repair houses} = \{h_2, h_4, h_5\}, \text{cheap houses} = \{h_1, h_2, h_3, h_4, h_5, h_6\}, \text{in good repair houses} = \{h_1, h_3, h_6\}, \text{in the green surroundings houses} = \{h_1, h_2, h_3, h_4, h_6\}\}$. 
Suppose that, Mr. X is interested to buy a house on the basis of his choice parameters beautiful, wooden, cheap, in the green surroundings, in good repair,

<table>
<thead>
<tr>
<th></th>
<th>beautiful</th>
<th>wooden</th>
<th>cheap</th>
<th>in good repair</th>
<th>in the green surroundings</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1</td>
<td>1</td>
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<tr>
<td>h2</td>
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<td>h3</td>
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<td>h4</td>
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<tr>
<td>h5</td>
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<tr>
<td>h6</td>
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</tbody>
</table>

The problem is to select the house which is most suitable with the choice parameters of Mr. X.
1st Method: Choice Value of an Object \( h_i \)

1. input the soft set \((F, E)\),
2. input the set \(P\) of choice parameters of Mr. X which is a subset of \(E\),
3. choose a soft subset \((F, P)\) of \((F, E)\).
4. Count the choice value \(c_i\) of an object \(h_i \in U\), given by
   \[ c_i = \sum_j h_{ij}, \]
   where \(h_{ij}\) are the entries in the table.
5. find \(k\), for which \(c_k = \max c_i\). Then \(h_k\) is the optimal choice object. If \(k\) has more than one value, then any one of them could be chosen by Mr. X by using his option.
An Application of SS-cont

<table>
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<tr>
<th></th>
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<th>in good repair</th>
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<th>Choose Value</th>
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<td>h2</td>
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<td>h6</td>
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<td>5</td>
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</tbody>
</table>

Decision: Mr. X can buy the house h6
2nd Method: Weighted Choice Value of an Object \( h_i \)

1. input the soft set \((F, E)\),
2. input the set \(P\) of choice parameters of Mr. X which is a subset of \(E\),
3. choose a soft subset \((F, P)\) of \((F, E)\).
4. Count the weighted choice value \( c_i \) of an object \( h_i \in U \), given by \( c_i = \sum_j d_{ij} \), where \( d_{ij} = w_j \times h_{ij} \), \( w_j \) is the \(j^{th}\) weight decided by Mr. X,
5. find \( k \), for which \( c_k = \max c_i \). Then \( h_k \) is the optimal choice object. If \( k \) has more than one value, then any one of them could be chosen by Mr. X by using his option.
<table>
<thead>
<tr>
<th>$U$</th>
<th>$e_1, w_1 = .8$</th>
<th>$e_2, w_2 = .3$</th>
<th>$e_4, w_4 = .9$</th>
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<td>$c_6 = 2.8$</td>
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N-soft sets and their decision making algorithms

Fatia Fatimah, Ded Rosadi, R. B. Fajriya Hakim, José Carlos R. Alcantud

Abstract In this paper, we motivate and introduce the concept of N-soft set as an extended soft set model. Some useful algebraic definitions and properties are given. We cite real examples that prove that N-soft sets are a cogent model for binary and non-binary evaluations in numerous kinds of decision making problems. Finally, we propose decision making procedures for N-soft sets.

Keywords N-soft set · Non-binary evaluation · Decision making · Choice value · Intersection and union

imprecision, or subjectivity. This paper expands the range of applications of one of the theories that can be used to deal with these characteristics, namely soft set theory. It was introduced by Molodtsov (1999), who also showed its applicability to various fields. Soft set does not require parameters specification. Instead, it accommodates all types of parameters as its benchmark. The parameters of a soft set can be numbers, words, sentences, functions, and so on. Thus, the soft set definition associates the pertinent attributes with information or knowledge about the elements in the universe.
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</table>

Confeccionada mediante los juicios de los críticos que figuran en el encabezamiento. El orden de las películas se establece según la media aritmética de las estrellas recibidas y del número de críticos que las valoran.
Example 5  We draw a reduced ‘cinema table’ from Fig. 1 and then transfer its information to the language of 6-soft set. Let $U$ be the universe of movies, $U = \{ u_1 = \text{Tarde para la ira}, u_2 = \text{Kubo y las dos cuerdas magicas}, u_3 = \text{Café society}, u_4 = \text{Neruda}, u_5 = \text{La estación de las mujeres} \}$. Let $E$ be the set of attributes ‘evaluations of movies by media,’ and $A \subseteq E$ be such that $A = \{ a_1 = \text{El mundo}, a_2 = \text{Guia del ocio}, a_3 = \text{Decine21.com}, a_4 = \text{ABC}, a_5 = \text{Cahiers}, a_6 = \text{Cineyteatro.es} \}$. In relation to these elements, a 6-soft set can be defined from Table 2, where five stars means ‘obra maestra’ (masterpiece), four stars means ‘muy buena’ (very good), three stars means ‘buena’ (good), two stars means ‘interesante’ (interesting), one star means ‘regular’ (average), and big dot means ‘mala’ (bad).
### Table 2  Information extracted from the real data in Fig. 1

<table>
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<tr>
<th></th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
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<td>***</td>
<td>.</td>
<td>**</td>
<td>***</td>
<td>***</td>
<td>**</td>
</tr>
</tbody>
</table>

It can be expressed as a 6-soft set (cf., Table 3)

### Table 3  The 6-soft set in Example 5

<table>
<thead>
<tr>
<th>$(F, A, 6)$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
<th>$a_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1$</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>$u_2$</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$u_3$</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$u_4$</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>$u_5$</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>
Algorithm 2  The algorithm of EWCVs.

1. Input $U = \{u_1, \ldots, u_p\}$ and $A = \{a_1, \ldots, a_q\}$, and a weight $w_j$ for each parameter $j$.
2. Input the $N$-soft set $(F, A, N)$, with $R = \{0, 1, \ldots, N - 1\}$, $N \in \{2, 3, \ldots\}$, so that $\forall u_i \in U, a_j \in A, \exists! r_{ij} \in R$.
3. For each $u_i$, compute its EWCV $\sigma_i^w = \sum_{j=1}^{q} w_j r_{ij}$.
4. Find all indices $k$ for which $\sigma_k^w = \max_{i=1,\ldots,p} \sigma_i^w$.
5. The solution is any $u_k$ from Step 4.

Extended weight choice values (EWCVs),

<table>
<thead>
<tr>
<th>$(F, A, 6)$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
<th>$a_6$</th>
<th>$\sigma_i^w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1 = 0.1$</td>
<td>0.4</td>
<td>0.9</td>
<td>0.9</td>
<td>0.4</td>
<td>0.5</td>
<td>0.4</td>
<td>3.5</td>
</tr>
<tr>
<td>$w_2 = 0.3$</td>
<td>0.4</td>
<td>1.5</td>
<td>0.9</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>4</td>
</tr>
<tr>
<td>$w_3 = 0.3$</td>
<td>0.4</td>
<td>0.6</td>
<td>0.9</td>
<td>0.3</td>
<td>0.4</td>
<td>0.4</td>
<td>3</td>
</tr>
<tr>
<td>$w_4 = 0.1$</td>
<td>0.5</td>
<td>0.9</td>
<td>0.6</td>
<td>0.3</td>
<td>0.4</td>
<td>0.3</td>
<td>3</td>
</tr>
<tr>
<td>$w_5 = 0.1$</td>
<td>0.3</td>
<td>0</td>
<td>0.6</td>
<td>0.3</td>
<td>0.3</td>
<td>0.2</td>
<td>1.7</td>
</tr>
</tbody>
</table>
In order to explain the concepts of fuzzy sets, the basic idea in classical set theory must be understood. In mathematics, the concept of classical set is very simple. A set is a collection of well-defined objects. These objects cover almost anything that can either belong or do not belong to the set.

The classical set $A$ in the universe $U$, $A \subseteq U$ is normally characterised by the function $\mu_A(x)$, which take value 1 or 0, indicating whether or not $x \in U$ is a member of $A$:

$$\mu_A(x) = \begin{cases} 1 & \text{for } x \in A \\ 0 & \text{for } x \notin A \end{cases} \quad (1)$$

Hence, $\mu_A(x) \in \{0, 1\}$. The function $\mu_A(x)$ as equation (1) takes only the value 1 or 0.
Ragin (2000) had a very simple explanation about fuzzy sets. He iterated that the basic idea behind fuzzy sets is to permit the scaling of membership scores and this allows partial or fuzzy membership.

A membership score of 1 indicates full membership in a set; a score close to 1 (e.g., 0.8 or 0.9) indicates strong but partial membership in a set; scores less than 0.5 but greater than 0 (e.g., 0.2 and 0.3) indicate that objects are more out than in a set, but still weak members of the set; a score of 0 indicates full non membership in the set.

Thus, fuzzy sets combine qualitative and quantitative assessment.
The fuzzy set theory can represent the uncertainty or vagueness inherent in the definition of linguistic variables (Zadeh, 1975).

Age is one of the examples of a linguistic variable whose values are words like very young, middle age, and very old.

Fuzzy set theory is becoming an alternative way in explaining the fuzzy phenomena in the real world.

Assume that the function $\mu_A(x)$ may take values in the interval $[0, 1]$. In this way, the concept of membership is not any more crisp, but become fuzzy in the sense of representing partial belonging or degree of membership (Bojadziev, 1999). A fuzzy set $R$ is defined by:

$$R = \{(x, \mu_R(x)) / x \in A, \mu_R(x) \in [0,1]\}$$

(2)

where $\mu_R(x)$ is a function called membership function; $\mu_R(x)$ specifies the grade or degree to which any element in $A$ belongs to the fuzzy set $R$. 
Seseorang dapat masuk dalam 2 himpunan yang berbeda, MUDA dan PAROBAYA, PAROBAYA dan TUA, dsb. Seberapa besar eksistensinya dalam himpunan tersebut dapat dilihat pada nilai keanggotaannya.

![Himpunan fuzzy untuk variabel Umur](image-url)

Admi Nazra Dept of Mathematics, Andalas University
The **fuzzy set (FS)** \( \mu \) over a set \( U \) is a set \( \{ (u, \mu(u)) | u \in U \} \) where \( \mu : U \rightarrow [0, 1] \).

\( \mu \) is called the membership function of \( X \)

\( \mu(u) \) is called the membership value of \( u \).

### Definition

Given two FSs \( \mu \) and \( \nu \) over \( U \).

- \( \mu^c := \{ (u, \mu(u)) | u \in U \}^c = \{ (u, \mu^c(u)) | u \in U \} \), with \( \mu^c(u) := 1 - \mu(u) \)
- \( \mu \cup \nu := \{ (u, \mu(u) \ast \nu(u)) | u \in U \} \), with \( \mu(u) \ast \nu(u) := \max\{\mu(u), \nu(u)\} \)
- \( \mu \cap \nu := \{ (u, \mu(u) \diamond \nu(u)) | u \in U \} \), with \( \mu(u) \diamond \nu(u) := \min\{\mu(u), \nu(u)\} \)
FUZZY SETS and Its Properties

Definition

Fuzzy set $\mu$ over $U$ is called

- The null FS, denoted by $\emptyset := \{(u, 0) | u \in U\}$
- The universal FS, denoted by $1 := \{(u, 1) | u \in U\}$

Proposition

Given two FSs $\mu$, $\nu$ and $w(u)$ over $U$.

- $(\mu^c)^c = \mu$
- $\mu \cup \nu = \nu \cup \mu$
- $\mu \cap \nu = \nu \cap \mu$
- $(\mu \cup \nu)^c = \nu^c \cap \mu^c$
- $(\mu \cap \nu)^c = \nu^c \cup \mu^c$
- $\mu \cap (\nu \cap w) = (\nu \cap \mu) \cap w$
- $\mu \cup (\nu \cup w) = (\nu \cup \mu) \cup w$
Definition (Maji, 2001)


A pair $(f_A, A)$ is called **a fuzzy soft set over $U$** if $f_A : A \rightarrow I^U$, $I^U$: the collection of all fuzzy sets over $U$. 


Admi Nazra Dept of Mathematics, Andalas University
Suppose that, Mr. X is interested to buy a house on the basis of his choice parameters beautiful, wooden, cheap, in the green surroundings, in good repair,

<table>
<thead>
<tr>
<th></th>
<th>beautiful</th>
<th>wooden</th>
<th>cheap</th>
<th>in good repair</th>
<th>in the green surroundings</th>
</tr>
</thead>
<tbody>
<tr>
<td>h1</td>
<td>(1;0.3)</td>
<td>0</td>
<td>(1;0.3)</td>
<td>(1;0.3)</td>
<td>(1;0.3)</td>
</tr>
<tr>
<td>h2</td>
<td>(1;0.7)</td>
<td>0</td>
<td>(1;0.3)</td>
<td>0</td>
<td>(1;0.3)</td>
</tr>
<tr>
<td>h3</td>
<td>(1;0.4)</td>
<td>0</td>
<td>(1;0.3)</td>
<td>(1;0.3)</td>
<td>(1;0.3)</td>
</tr>
<tr>
<td>h4</td>
<td>(1;0.9)</td>
<td>(1;0.3)</td>
<td>(1;0.3)</td>
<td>0</td>
<td>(1;0.3)</td>
</tr>
<tr>
<td>h5</td>
<td>(1;0.1)</td>
<td>(1;0.3)</td>
<td>(1;0.3)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>h6</td>
<td>(1;0.3)</td>
<td>(1;0.3)</td>
<td>(1;0.3)</td>
<td>(1;0.3)</td>
<td>(1;0.3)</td>
</tr>
</tbody>
</table>

The problem is to select the house which is most suitable with the choice parameters of Mr. X.
Suppose that Mr. X is interested to buy a house from among the set of houses \( U = \{ h_1; h_2; h_3 \} \) on the basis of the set \( E = \{ e_1 = \text{cheap house}, e_2 = \text{beautiful house}, e_3 = \text{green surroundings}, e_4 = \text{good location house} \} \) of selection criteria called the parameters.

Suppose Mr. X is going to buy the house on his own preference weightage to the selection criteria.

Now, to get the recent market information, that is, the performance evaluation matrix we construct the fuzzy soft sets, \((F_1,E), (F_2,E), (F_3,E)\) over the universe \( U \).
(\(F_1, E\)) = \begin{pmatrix}
  e_1 & 0.7 & 0.6 & 0.3 \\
  e_2 & 0.5 & 0.4 & 0.7 \\
  e_3 & 0.7 & 0.5 & 0.5 \\
  e_4 & 0.4 & 0.7 & 0.8 \\
\end{pmatrix},

(\(F_2, E\)) = \begin{pmatrix}
  e_1 & 0.4 & 0.7 & 0.3 \\
  e_2 & 0.8 & 0.4 & 0.5 \\
  e_3 & 0.9 & 0.5 & 0.6 \\
  e_4 & 0.5 & 0.7 & 0.9 \\
\end{pmatrix},

(\(F_3, E\)) = \begin{pmatrix}
  e_1 & 0.2 & 0.9 & 0.4 \\
  e_2 & 0.5 & 0.7 & 0.4 \\
  e_3 & 0.4 & 0.7 & 0.8 \\
  e_4 & 0.6 & 0.4 & 0.8 \\
\end{pmatrix}.
Then by taking the average of the above three fuzzy soft sets, we get the performance evaluation matrix (or recent market survey information) as follow:

\[
R = \begin{pmatrix}
  e_1 & 0.433 & 0.733 & 0.333 \\
  e_2 & 0.600 & 0.500 & 0.533 \\
  e_3 & 0.667 & 0.567 & 0.633 \\
  e_4 & 0.500 & 0.600 & 0.833 \\
\end{pmatrix}.
\]

Hence

\[
R^T = \begin{pmatrix}
  e_1 & e_2 & e_3 & e_4 \\
  h_1 & 0.433 & 0.600 & 0.667 & 0.500 \\
  h_2 & 0.733 & 0.500 & 0.567 & 0.600 \\
  h_3 & 0.333 & 0.533 & 0.633 & 0.833 \\
\end{pmatrix}.
\]
Next, suppose that the preference weightage of $Mr \cdot X$ to the different selection criteria is given by the following table:

$$W = \begin{bmatrix}
e_1 & e_2 & e_3 & e_4 \\
0.3 & 0.2 & 0.1 & 0.4
\end{bmatrix} \quad \text{such that } \sum_{i=1}^{4} = W_j \leq 1$$

Table 2

Thus, to get the comprehensive decision matrix $D$ for $Mr \cdot X$, we multiply $R^T$ by the preference weightage matrix and get matrix $D$ as follows:
FUZZY SOFT SETS-ex

$D = \begin{pmatrix}
    h_1 & e_1 & 0.129 & 0.120 & 0.067 & 0.200 \\
    h_2 & e_2 & 0.220 & 0.100 & 0.057 & 0.240 \\
    h_3 & e_3 & 0.010 & 0.107 & 0.063 & 0.330
\end{pmatrix}$

The comparison table of the above comprehensive decision matrix is:

<table>
<thead>
<tr>
<th>$U$</th>
<th>$h_1$</th>
<th>$h_2$</th>
<th>$h_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_1$</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$h_2$</td>
<td>2</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$h_3$</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 3
Next, we compute the row-sum, column-sum from the comprehensive decision matrix and the score for each house $h_i, i = 1, 2, 3$ as follows:

<table>
<thead>
<tr>
<th></th>
<th>Row-sum</th>
<th>Column-sum</th>
<th>Score value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_1$</td>
<td>9</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>$h_2$</td>
<td>7</td>
<td>9</td>
<td>-2</td>
</tr>
<tr>
<td>$h_3$</td>
<td>8</td>
<td>8</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4: Scor table.

Obviously from the above table, we notice that the maximum score is 2, which is related to the house $h_1$. Therefore, the house $h_1$ is the best choice for $M_r \cdot X$. 
<table>
<thead>
<tr>
<th>Steps</th>
<th>An algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step I</td>
<td>Input the performance evaluation of houses by different counseling agencies as matrix.</td>
</tr>
<tr>
<td>Step II</td>
<td>Find the average of the corresponding entries of all the matrices in Step (I).</td>
</tr>
<tr>
<td>Step III</td>
<td>Multiply the weightage of different houses of the guardins to the corresponding entries of each row to get the comprehensive decision matrix.</td>
</tr>
<tr>
<td>Step IV</td>
<td>Formulate the comparison table.</td>
</tr>
<tr>
<td>Step V</td>
<td>Find the row-sums and column-sums of the comparison table.</td>
</tr>
<tr>
<td>Step VI</td>
<td>Obtain the score for each product and the product with maximum score is recommended as the best choice.</td>
</tr>
</tbody>
</table>
Intuitionistic Fuzzy Soft Set/ IFSS

Definition (Atanassov, 1986)

Let \( U \) be a set. An intuitionistic fuzzy set \( X \) over the set \( U \) is defined as

\[
X = \{(x, \mu_X(z), \gamma_X(z)) | z \in U\}
\]

where \( \mu_X : U \rightarrow [0, 1] \) and \( \gamma_X : U \rightarrow [0, 1] \), such that

\[
0 \leq \mu_X(z) + \gamma_X(z) \leq 1 \text{ for all } z \in U.
\]

\( \mu_X(z) \) is called the membership value of \( z \)

\( \gamma_X(z) \) is the non-membership value of \( z \).

Definition (Maji et. al, 2004)

Let \( U \) be a set, \( E \) is the set of parameters, \( A \subset E \). A pair \((F, A)\) is called an intuitionistic fuzzy soft set over \( U \), where \( F \) is a mapping given by \( F : A \rightarrow \mathcal{IF}(U) \), \( \mathcal{IF}(U) \) is a collection of all intuitionistic fuzzy sets over \( U \).
Let us consider an intuitionistic fuzzy soft set $\omega = (F, A)$ which describes the "attractiveness of houses" that Mr. X is considering for purchase.

Suppose there are six houses in the domain $U = \{h_1; h_2; h_3; h_4; h_5; h_6\}$ under consideration, and $A = \{e_1; e_2; e_3; e_4; e_5; e_6\}$ is a set of decision parameters. The $e_i$ ($i = 1; 2; 3; 4; 5; 6$) stand for the parameters: modern, cheap, beautiful, large, convenient traffic and in green surroundings, respectively.

Table 1 gives the tabular representation of the intuitionistic fuzzy soft set $\omega = (F, A)$.
Table 1

<table>
<thead>
<tr>
<th>$U$</th>
<th>$\varepsilon_1$</th>
<th>$\varepsilon_2$</th>
<th>$\varepsilon_3$</th>
<th>$\varepsilon_4$</th>
<th>$\varepsilon_5$</th>
<th>$\varepsilon_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_1$</td>
<td>(0.3, 0.5)</td>
<td>(0.6, 0.3)</td>
<td>(0.6, 0.3)</td>
<td>(0.6, 0.3)</td>
<td>(0.3, 0.5)</td>
<td>(0.5, 0.2)</td>
</tr>
<tr>
<td>$h_2$</td>
<td>(0.8, 0.0)</td>
<td>(0.8, 0.1)</td>
<td>(0.8, 0.0)</td>
<td>(0.6, 0.2)</td>
<td>(0.4, 0.1)</td>
<td>(0.6, 0.1)</td>
</tr>
<tr>
<td>$h_3$</td>
<td>(0.5, 0.4)</td>
<td>(0.5, 0.4)</td>
<td>(0.2, 0.6)</td>
<td>(0.2, 0.6)</td>
<td>(0.5, 0.3)</td>
<td>(0.2, 0.1)</td>
</tr>
<tr>
<td>$h_4$</td>
<td>(0.2, 0.7)</td>
<td>(0.2, 0.6)</td>
<td>(0.0, 0.9)</td>
<td>(0.0, 0.9)</td>
<td>(0.2, 0.4)</td>
<td>(0.1, 0.7)</td>
</tr>
<tr>
<td>$h_5$</td>
<td>(0.7, 0.0)</td>
<td>(0.8, 0.1)</td>
<td>(0.8, 0.0)</td>
<td>(0.7, 0.1)</td>
<td>(0.5, 0.3)</td>
<td>(0.7, 0.1)</td>
</tr>
<tr>
<td>$h_6$</td>
<td>(0.8, 0.0)</td>
<td>(0.8, 0.1)</td>
<td>(0.5, 0.4)</td>
<td>(0.7, 0.1)</td>
<td>(0.8, 0.1)</td>
<td>(0.5, 0.1)</td>
</tr>
</tbody>
</table>

If we choose $\lambda = \text{topbottom}_\varpi$ where $\text{topbottom}_\varpi: A \rightarrow [0,1] \times [0,1]$ defined by $\mu_{\text{topbottom}_\varpi}(\varepsilon) = \max\{\mu_F(\varepsilon)(x) | x \in U\}$ and $\nu_{\text{topbottom}_\varpi}(\varepsilon) = \min\{\mu_F(\varepsilon)(x) | x \in U\}$ for all $\varepsilon \in A$, then $\text{topbottom}_\varpi = \{(\varepsilon_1, 0.8, 0), (\varepsilon_2, 0.8, 0.1), (\varepsilon_3, 0.8, 0), (\varepsilon_4, 0.7, 0.1), (\varepsilon_5, 0.8, 0.1), (\varepsilon_6, 0.7, 0.1)\}$. 
By using Algorithm 1, we obtain the level soft set \( L(\varpi; \text{topbottom}_{\varpi}) \) with the choice values with tabular representation as in Table 2.

**Table 1**

<table>
<thead>
<tr>
<th>( U )</th>
<th>( \varepsilon_1 )</th>
<th>( \varepsilon_2 )</th>
<th>( \varepsilon_3 )</th>
<th>( \varepsilon_4 )</th>
<th>( \varepsilon_5 )</th>
<th>( \varepsilon_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_1 )</td>
<td>(0.3, 0.5)</td>
<td>(0.6, 0.3)</td>
<td>(0.6, 0.3)</td>
<td>(0.6, 0.3)</td>
<td>(0.3, 0.5)</td>
<td>(0.5, 0.2)</td>
</tr>
<tr>
<td>( h_2 )</td>
<td>(0.8, 0.0)</td>
<td>(0.8, 0.1)</td>
<td>(0.8, 0.0)</td>
<td>(0.6, 0.2)</td>
<td>(0.4, 0.1)</td>
<td>(0.6, 0.1)</td>
</tr>
<tr>
<td>( h_3 )</td>
<td>(0.5, 0.4)</td>
<td>(0.5, 0.4)</td>
<td>(0.2, 0.6)</td>
<td>(0.2, 0.6)</td>
<td>(0.5, 0.3)</td>
<td>(0.2, 0.1)</td>
</tr>
<tr>
<td>( h_4 )</td>
<td>(0.2, 0.7)</td>
<td>(0.2, 0.6)</td>
<td>(0.0, 0.9)</td>
<td>(0.0, 0.9)</td>
<td>(0.2, 0.4)</td>
<td>(0.1, 0.7)</td>
</tr>
<tr>
<td>( h_5 )</td>
<td>(0.7, 0.0)</td>
<td>(0.8, 0.1)</td>
<td>(0.8, 0.0)</td>
<td>(0.7, 0.1)</td>
<td>(0.5, 0.3)</td>
<td>(0.7, 0.1)</td>
</tr>
<tr>
<td>( h_6 )</td>
<td>(0.8, 0.0)</td>
<td>(0.8, 0.1)</td>
<td>(0.5, 0.4)</td>
<td>(0.7, 0.1)</td>
<td>(0.8, 0.1)</td>
<td>(0.5, 0.1)</td>
</tr>
</tbody>
</table>

**Table 2**

<table>
<thead>
<tr>
<th>( U )</th>
<th>( \varepsilon_1 )</th>
<th>( \varepsilon_2 )</th>
<th>( \varepsilon_3 )</th>
<th>( \varepsilon_4 )</th>
<th>( \varepsilon_5 )</th>
<th>( \varepsilon_6 )</th>
<th>Choice value ( (c_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_1 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( c_1 = 0 )</td>
</tr>
<tr>
<td>( h_2 )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( c_2 = 3 )</td>
</tr>
<tr>
<td>( h_3 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( c_3 = 0 )</td>
</tr>
<tr>
<td>( h_4 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( c_4 = 0 )</td>
</tr>
<tr>
<td>( h_5 )</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>( c_5 = 4 )</td>
</tr>
<tr>
<td>( h_6 )</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>( c_6 = 4 )</td>
</tr>
</tbody>
</table>
Algorithm 1.

1. Input the (result) intuitionistic fuzzy soft set $\varpi = (F, A)$.
2. Input a threshold intuitionistic fuzzy set $\lambda : A \rightarrow [0, 1] \times [0, 1]$ for decision making.
3. Compute the level soft set $L(\varpi; \lambda)$.
4. Present the level soft set $L(\varpi; \lambda)$ in tabular form and compute the choice value $c_i$ of $x_i$, $\forall i$.
5. The optimal decision is to select $x_k$ if $c_k = \max_i c_i$.
6. If $k$ has more than one value, then any one of $x_k$ may be chosen.
Definition (Torra, 2010)

Let $U$ be a set. A **Hesitant fuzzy set** $X$ over the set $U$ is defined as

$$X = \{(x, h(z)) | z \in U\}$$

where $h: U \rightarrow \mathcal{P}([0, 1])$

$h(z)$ is called the membership value of $z$

$\mathcal{P}([0, 1])$ is a collection of all subsets of $[0,1]$.

$h(z)$ is called a hesitant fuzzy element of $z \in U$.

Definition (Babitha & John, 2013)

Let $U$ be a set, $E$ is the set of parameters, $A \subset E$. A pair $(F,A)$ is called an **Hesitant fuzzy soft set** over $U$, where $F$ is a mapping given by $F: A \rightarrow \mathcal{HF}(U)$, $\mathcal{HF}(U)$ is a collection of all Hesitant fuzzy sets over $U$. 

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**Definition (Xia & Xu, 2011)**

For a hesitant fuzzy element \( h(x) \), \( s(h) = \frac{1}{l(h)} \sum_{\gamma \in h(x)} \gamma \) is called the score function of \( h(x) \) where \( l(h) \) denotes number of values in \( h(x) \).

**Definition (Babitha & John, 2013)**

Let \((F, A)\) denotes hesitant fuzzy soft set, Then the fuzzy soft set \((F_S, A)\) in which each entries in the fuzzy set \( F_S(e) \) is the score function of the respective entries in the hesitant fuzzy set \( F(e) \) is called as score matrix.

**Definition (Babitha & John, 2013)**

The table obtained by calculating the average of \( F_S(e_i) \) for each \( u_j \) is called as decision table.
Algorithm

1. Input the hesitant fuzzy soft set \((F, A)\)
2. Obtain the score matrix \((F_S, A)\) corresponds to \((F, A)\)
3. Calculate the average of \(F_S(e_i)\) for each \(u_j\) and let it be denoted as \(a_j\).
   This is the decision table
4. Select the optimal alternative \(u_k\) if \(a_k = \max_j a_j\)
5. If \(k\) has more than one value then any one of \(u_k\) may be chosen.

Remark 4.5 In decision making problems further representational capability can be added by associating with each parameter \(e_i\) a value \(w_i \in [0, 1]\) called its weight. In the case of multi-criteria decision making, these weights can be used to represent the different importance of the concerned criteria. In this case there is a small change in the above algorithm. In step 3 instead of average we take weighted average

\[
\frac{\sum_{i=1}^{n} F_S(e_i) w_i}{n}
\]

and follows the next step.
Let us consider a decision-making problem of allocating a particular job to the best possible person who fulfills the requirements of the job. Selection is done by the interview board consisting of three members. Let $U = \{u_1, u_2, u_3, u_4\}$ be a crisp set of four persons for the job. Let $A = \{\text{enterprising}, \text{confident}, \text{willing to take risks}, \text{hardworking}\}$ be the set of parameters which represents the criteria for the problem. Let $A$ can be represented as $A = \{e_1, e_2, e_3, e_4\}$. The problem is the selection of the best person who satisfies the criteria to the utmost extent.

All the available information on these candidates can be characterized by hesitant fuzzy soft set $(F, A)$. The tabular representation of the hesitant fuzzy soft set $(F, A)$ is shown in Table 1. In Table 1, we can see that the evaluation for an alternative to satisfy a criterion is represented by hesitant fuzzy set representing the grades given by the three interviewers.
### Table 1

<table>
<thead>
<tr>
<th></th>
<th>$u_1$</th>
<th>$u_2$</th>
<th>$u_3$</th>
<th>$u_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_1$</td>
<td>{0.6,0.70,0.8}</td>
<td>{0.3,0.4,0.45}</td>
<td>{0.5,0.6,0.7}</td>
<td>{0.7,0.8,0.8}</td>
</tr>
<tr>
<td>$e_2$</td>
<td>{0.7,0.8,0.85}</td>
<td>{0.6,0.7,0.65}</td>
<td>{0.7,0.8,0.65}</td>
<td>{0.6,0.7,0.8}</td>
</tr>
<tr>
<td>$e_3$</td>
<td>{0.76,0.82,0.65}</td>
<td>{0.76,0.7,0.8}</td>
<td>{0.81,0.66,0.9}</td>
<td>{0.9,0.8,0.9}</td>
</tr>
<tr>
<td>$e_4$</td>
<td>{0.8,0.82,0.88}</td>
<td>{0.74,0.68,0.52}</td>
<td>{0.56,0.7,0.68}</td>
<td>{0.76,0.7,0.8}</td>
</tr>
</tbody>
</table>
Then the score matrix \((F_S, A)\) corresponds to \((F, A)\) given in the table 1 is as follows:

<table>
<thead>
<tr>
<th></th>
<th>(u_1)</th>
<th>(u_2)</th>
<th>(u_3)</th>
<th>(u_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e_1)</td>
<td>0.7</td>
<td>0.3833</td>
<td>0.6</td>
<td>0.766667</td>
</tr>
<tr>
<td>(e_2)</td>
<td>0.7833</td>
<td>0.65</td>
<td>0.7166</td>
<td>0.70667</td>
</tr>
<tr>
<td>(e_3)</td>
<td>0.7433</td>
<td>0.7533</td>
<td>0.79</td>
<td>0.866667</td>
</tr>
<tr>
<td>(e_4)</td>
<td>0.8333</td>
<td>0.6466</td>
<td>0.64667</td>
<td>0.7533</td>
</tr>
</tbody>
</table>

Table 2

The decision table for each person \(u_j\) obtained as follows:

<table>
<thead>
<tr>
<th>(a_j)</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_1)</td>
<td>.76497</td>
</tr>
<tr>
<td>(a_2)</td>
<td>.6083</td>
</tr>
<tr>
<td>(a_3)</td>
<td>.688</td>
</tr>
<tr>
<td>(a_4)</td>
<td>.7733</td>
</tr>
</tbody>
</table>

Table 3
Definition (Beg and Rashid, 2014)

Given a set \( U \). Suppose \( \mu \) and \( \mu' \) are functions applied to \( U \) return subsets of \([0, 1]\) where \( \mu(u) \) and \( \mu'(u) \) are sets of some values in \([0, 1]\).

An hesitant intuitionistic fuzzy set (HIFS) on \( U \) is a set

\[
\tilde{X} = \{ < u, \mu(u), \mu'(u) > | u \in U \}.
\]

Values \( \mu(u) \) and \( \mu'(u) \) denote the possible membership degrees and non-membership degrees of the element \( u \in U \) to the set \( \tilde{X} \), which satisfy

\[
\max \{ \mu(u) \} + \min \{ \mu'(u) \} \leq 1
\]

and

\[
\min \{ \mu(u) \} + \max \{ \mu'(u) \} \leq 1
\]
For simplification, \( \tilde{\mu}(u) = (\mu(u), \mu'(u)) \) is called an **hesitant intuitionistic fuzzy element (HIFE)**, with

\[
\mu(u) = \{\alpha_1, \ldots, \alpha_m | \alpha_i \in [0, 1]\}
\]

and

\[
\mu'(u) = \{\alpha'_1, \ldots, \alpha'_n | \alpha'_i \in [0, 1]\}.
\]

The **HIFS** \( \tilde{X} \) can be written as

\[
\tilde{X} = \{< u, \tilde{\mu}(u) > | u \in U\}.
\]

The set of all HIFSs on \( U \) is denoted by \( \tilde{I}\mathcal{H}(U) \).
An HIFS $\tilde{X} = \{< u, \mu(u), \mu'(u) > | u \in U \}$ is called:

i) The **null HIFS**, denoted by $\tilde{\emptyset} = \{< u, \tilde{\mu}(u) > | u \in U \}$, if
$\tilde{\mu}(u) = (\mu(u), \mu'(u)) = (\{0\}, \{1\})$.

ii) The **universal HIFS**, denoted by $\tilde{1} = \{< u, \tilde{\mu}(u) > | u \in U \}$, if
$\tilde{\mu}(u) = (\mu(u), \mu'(u)) = (\{1\}, \{0\})$. 
The operations of HIFEs

Definition

Let \( \tilde{\mu}_i(u) = (\mu_i(u), \mu'_i(u)) \) be HIFEs with
\[
\mu_1(u) = \{\alpha_1, \ldots, \alpha_m | \alpha_i \in [0, 1]\}, \quad \mu'_1(u) = \{\alpha'_1, \ldots, \alpha'_n | \alpha'_i \in [0, 1]\}.
\]
\[
\mu_2(u) = \{\beta_1, \ldots, \beta_p | \beta_i \in [0, 1]\}, \quad \mu'_2(u) = \{\beta'_1, \ldots, \beta'_q | \beta'_i \in [0, 1]\}.
\]

We define the following HIFE operations:

i) \( \tilde{\mu}^c(u) = (\mu'(u), \mu(u)) \)

ii) \( \tilde{\mu}_1(u) \cup \tilde{\mu}_2(u) = \)
\[
\left( \bigcup_{\alpha_i \in \mu_1(u)} \bigg\{ \max\{\alpha_i, \beta_j\} \bigg\}, \bigcup_{\alpha'_i \in \mu'_1(u)} \bigg\{ \min\{\alpha'_i, \beta'_j\} \bigg\} \right)_{\beta_j \in \mu_2(u)}.
\]

iii) \( \tilde{\mu}_1(u) \cap \tilde{\mu}_2(u) = \)
\[
\left( \bigcup_{\alpha_i \in \mu_1(u)} \bigg\{ \min\{\alpha_i, \beta_j\} \bigg\}, \bigcup_{\alpha'_i \in \mu'_1(u)} \bigg\{ \max\{\alpha'_i, \beta'_j\} \bigg\} \right)_{\beta'_j \in \mu'_2(u)}.
\]
The operations of HIFSs (Nazra et. al, 2018)

Definition

Given two HIFSs $\tilde{X} = \{ < u, \tilde{\mu}(u) > | u \in U \}$, $\tilde{Y} = \{ < u, \tilde{\nu}(u) > | u \in U \}$ on $U$.

We define the following operations

i) $\tilde{X}^c = \{ < u, \tilde{\mu}^c(u) > | u \in U \}$.

ii) $\tilde{X} \cup \tilde{Y} = \{ < u, \tilde{\mu}(u) \cup \tilde{\nu}(u) > | u \in U \}$.

iii) $\tilde{X} \cap \tilde{Y} = \{ < u, \tilde{\mu}(u) \cap \tilde{\nu}(u) > | u \in U \}$. 
Proposition

Let \( \tilde{\mu}_i \) be HIFEs. Then the following hold.

i) \( (\tilde{\mu}_i^c(u))^c = \mu_i(u) \).

ii) \( \tilde{\mu}_1(u) \cup \tilde{\mu}_2(u) = \tilde{\mu}_2(u) \cup \tilde{\mu}_1(u) \).

iii) \( \tilde{\mu}_1(u) \cap \tilde{\mu}_2(u) = \tilde{\mu}_2(u) \cap \tilde{\mu}_1(u) \).

iv) \( (\tilde{\mu}_1(u) \cup \tilde{\mu}_2(u))^c = \tilde{\mu}_1^c(u) \cap \tilde{\mu}_2^c(u) \).

v) \( (\tilde{\mu}_1(u) \cap \tilde{\mu}_2(u))^c = \tilde{\mu}_1^c(u) \cup \tilde{\mu}_2^c(u) \).

vi) \( \tilde{\mu}_1(u) \cap (\tilde{\mu}_2(u) \cap \tilde{\mu}_3(u)) = (\tilde{\mu}_1(u) \cap \tilde{\mu}_2(u)) \cap \tilde{\mu}_3(u) \).

vii) \( \tilde{\mu}_1(u) \cup (\tilde{\mu}_2(u) \cup \tilde{\mu}_3(u)) = (\tilde{\mu}_1(u) \cup \tilde{\mu}_2(u)) \cup \tilde{\mu}_3(u) \).

viii) \( \tilde{\mu}_1(u) \cup (\tilde{\mu}_2(u) \cap \tilde{\mu}_3(u)) = (\tilde{\mu}_1(u) \cup \tilde{\mu}_2(u)) \cap (\tilde{\mu}_1(u) \cup \tilde{\mu}_3(u)) \).

ix) \( \tilde{\mu}_1(u) \cap (\tilde{\mu}_2(u) \cup \tilde{\mu}_3(u)) = (\tilde{\mu}_1(u) \cap \tilde{\mu}_2(u)) \cup (\tilde{\mu}_1(u) \cap \tilde{\mu}_3(u)) \).
Proposition

Let $\tilde{X}$, $\tilde{Y}$, $\tilde{Z}$ be HIFSs on $U$.

i) $(\tilde{X}^c)^c = \tilde{X}$.

ii) $\tilde{X} \cup \tilde{Y} = \tilde{Y} \cup \tilde{X}$.

iii) $\tilde{X} \cap \tilde{Y} = \tilde{Y} \cap \tilde{X}$.

iv) $(\tilde{X} \cup \tilde{Y})^c = \tilde{X}^c \cap \tilde{Y}^c$.

v) $(\tilde{X} \cap \tilde{Y})^c = \tilde{X}^c \cup \tilde{Y}^c$.

vi) $\tilde{X} \cup (\tilde{Y} \cap \tilde{Z}) = (\tilde{X} \cup \tilde{Y}) \cap (\tilde{X} \cup \tilde{Z})$.

vii) $\tilde{X} \cap (\tilde{Y} \cup \tilde{Z}) = (\tilde{X} \cap \tilde{Y}) \cup (\tilde{X} \cap \tilde{Z})$. 

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Generalized Hesitant Intuitionistic Fuzzy Soft Set (GHIFSS) (Nazra et. al, 2018)

**Definition**

Given $\tilde{IH}(U)$ the collection of all HIFSs on $U$, $\tilde{F}: E \rightarrow \tilde{IH}(U)$ and $\alpha$ fuzzy set on $E$.

A pair $(\tilde{F}_\alpha, E)$ is called a **GHIFSS over $U$**, where

$\tilde{F}_\alpha: E \rightarrow \tilde{IH}(U) \times [0, 1],$

$\tilde{F}_\alpha(e) = (\tilde{F}(e), \alpha(e)) = (\{(u, \mu_e(u), \mu'_e(u))| u \in U\}, \alpha(e))$

$\in \tilde{IH}(U) \times [0, 1]$ and

$(\tilde{F}_\alpha, E) = \{< e, \tilde{F}(e), \alpha(e) >\}$.

**Definition**

The **complement** of an GHIFSS $(\tilde{F}_\alpha, E)$ is defined as

$(\tilde{F}_\alpha, E)^c = \{< e, \tilde{F}^c(e), \alpha^c(e) >\}$. 

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Admi Nazra Dept of Mathematics, Andalas University
The operations of GHIFSSs (Nazra et. al, 2018)

Definition

Given two GHIFSSs \((\tilde{F}_\alpha, A)\) and \((\tilde{G}_\beta, B)\) over \(U\). We define two operations on such GHIFSSs as follows:

1. \((\tilde{F}_\alpha, A) \wedge (\tilde{G}_\beta, B) := (\tilde{T}_\gamma, A \times B)\)
   \[\tilde{T}_\gamma : A \times B \to \tilde{IH}(U) \times [0, 1]\]
   \[\gamma : A \times B \to [0, 1]\]
   which are defined by
   \[\tilde{T}(a, b) := \tilde{F}(a) \cap \tilde{G}(b),\]
   \[\gamma(a, b) = \alpha(a) \circ \beta(b).\]

2. \((\tilde{F}_\alpha, A) \vee (\tilde{G}_\beta, B) := (\tilde{N}_\psi, A \times B),\)
   \[\tilde{N}_\psi : A \times B \to \tilde{IH}(U) \times [0, 1]\]
   \[\psi : A \times B \to [0, 1]\]
   which are defined by
   \[\tilde{N}(a, b) := \tilde{F}(a) \cup \tilde{G}(b),\]
   \[\psi(a, b) = \alpha(a) \star \beta(b).\]
**Theorem**

Given two GHIFSSs \((\tilde{F}_\alpha, A)\) and \((\tilde{G}_\beta, B)\) over \(U\). Then the following De Morgan’s laws hold.

i) \(((\tilde{F}_\alpha, A) \land (\tilde{G}_\beta, B))^c = (\tilde{F}_\alpha, A)^c \lor (\tilde{G}_\beta, B)^c\)

ii) \(((\tilde{F}_\alpha, A) \lor (\tilde{G}_\beta, B))^c = (\tilde{F}_\alpha, A)^c \land (\tilde{G}_\beta, B)^c\)

**Proof.**

By using the previous definition on GHIFSSs, proposition on HIFSs and Fuzzy sets;

i) \(((\tilde{F}_\alpha, A) \land (\tilde{G}_\beta, B))^c = (\tilde{T}_\gamma, A \times B)^c\).

\[\tilde{T}^c(a, b) = (\tilde{F}(a) \cap \tilde{G}(b))^c = \tilde{F}(a)^c \cup \tilde{G}(b)^c = \tilde{N}(a, b),\]

\[\gamma^c(a, b) = (\alpha(a) \diamond \beta(b))^c = \alpha^c(a) \star \beta^c(b) = \psi(a, b).\]

where \((\tilde{N}_\psi, A \times B) = \{(e, \tilde{N}(e), \psi(e))\} = (\tilde{F}_\alpha, A)^c \lor (\tilde{G}_\beta, B)^c\).
Theorem

Given three GHIFSSs $(\tilde{F}_\alpha, A)$, $(\tilde{K}_\beta, B)$ and $(\tilde{G}_\gamma, C)$ over $U$. Then the following associative law holds:

i) $$(\tilde{F}_\alpha, A) \land ((\tilde{K}_\beta, B) \land (\tilde{G}_\gamma, C)) = ((\tilde{F}_\alpha, A) \land (\tilde{K}_\beta, B)) \land (\tilde{G}_\gamma, C)$$

ii) $$(\tilde{F}_\alpha, A) \lor ((\tilde{K}_\beta, B) \lor (\tilde{G}_\gamma, C)) = ((\tilde{F}_\alpha, A) \lor (\tilde{K}_\beta, B)) \lor (\tilde{G}_\gamma, C)$$

Proof.

The proof of this theorem follows from the previous definition on GHIFSSs, proposition on HIFSs and Fuzzy sets.
The properties of IHFSSs-cont (Nazra et. al, 2018)

Note that the distributive law of GHIFSS does not hold because

\[ A \times (B \times C) \neq (A \times B) \times (A \times C). \]

\[(\tilde{F}_\alpha, A) \land ((\tilde{K}_\beta, B) \lor (\tilde{G}_\gamma, C)) \neq \]
\[ ((\tilde{F}_\alpha, A) \land (\tilde{K}_\beta, B)) \lor ((\tilde{F}_\alpha, A) \land (\tilde{G}_\gamma, C)), \]

\[(\tilde{F}_\alpha, A) \lor ((\tilde{K}_\beta, B) \land (\tilde{G}_\gamma, C)) \neq \]
\[ ((\tilde{F}_\alpha, A) \lor (\tilde{K}_\beta, B)) \land ((\tilde{F}_\alpha, A) \lor (\tilde{G}_\gamma, C)). \]
The operations of GHIFSSs-cont (Nazra et. al, 2018)

Definition

Given two GHIFSSs \((\tilde{F}_\alpha, A)\) and \((\tilde{G}_\beta, B)\) over \(U\).

The \textbf{Union} of \((\tilde{F}_\alpha, A)\) and \((\tilde{G}_\beta, B)\), denoted by \((\tilde{F}_\alpha, A) \hat{\cup} (\tilde{G}_\beta, B) =: (\tilde{J}_\gamma, C)\) is an GHIFSS, where \(C = A \cup B\), and \(\forall \ c \in C\),

\[
\tilde{J}(c) = \left\{ \begin{array}{ll}
\tilde{F}(c), & \text{if } c \in A - B, \\
\tilde{G}(c), & \text{if } c \in B - A \\
\tilde{F}(c) \cup \tilde{G}(c), & \text{if } c \in A \cap B.
\end{array} \right.
\]

\[
\gamma(c) = \left\{ \begin{array}{ll}
\alpha(c), & \text{if } c \in A - B, \\
\beta(c), & \text{if } c \in B - A \\
\alpha(c) \star \beta(c), & \text{if } c \in A \cap B.
\end{array} \right.
\]
The operations of GHIFSSs-cont (Nazra et. al, 2018)

Definition

The **Intersection** of \((\tilde{F}_\alpha, A)\) and \((\tilde{G}_\beta, B)\), denoted by \((\tilde{F}_\alpha, A) \hat{\cap} (\tilde{G}_\beta, B) =: (\tilde{J}_\gamma, C)\) is an GHIFSS, where \(C = A \cap B \neq \emptyset\), and \(\forall c \in C\),

\[\tilde{J}(c) = \tilde{F}(c) \cap \tilde{G}(c)\text{ and } \gamma(c) = \alpha(c) \diamond \beta(c).\]

Definition

A GHIFSS \((\tilde{F}_\alpha, A) = \{< e, \tilde{F}(e), \alpha(e) >\}\) is called:

i) **The null GHIFSS**, denoted by \((\tilde{0}, A)\), if \(\tilde{F}(e) := \tilde{0}\), \(\alpha(e) := 0\).

ii) **The universal GHIFSS**, denoted by \((\tilde{1}, A)\), if \(\tilde{F}(e) := \tilde{1}\), \(\alpha(e) := 1\).
Note that if $A \neq B$ then the De Morgan’s law is not satisfied for the union and the intersection of GHIFSSs.

**Theorem**

*Given two GHIFSSs over $U$, $(\tilde{F}_\alpha, A)$ and $(\tilde{G}_\beta, A)$. Then*

i) \( ((\tilde{F}, A) \hat{\cup}(\tilde{G}_\beta, B))^c = (\tilde{F}_\alpha, A)^c \hat{\cap}(\tilde{G}_\beta, A)^c \)

ii) \( ((\tilde{F}, A) \hat{\cap}(\tilde{G}_\beta, B))^c = (\tilde{F}_\alpha, A)^c \hat{\cup}(\tilde{G}_\beta, A)^c. \)

**Theorem**

*Given $\tilde{G}H$ the collection of all GHIFSSs over $U$, then the following holds*

- $\tilde{G}H$ is closed over operations $\hat{\cap}$ and $\hat{\cup}$
- $\tilde{G}H$ satisfies the associative law over operations $\hat{\cap}$ and $\hat{\cup}$
- there are GHIFSSs $(\tilde{1}_1, A)$ and $(\tilde{\emptyset}, A)$ such that $(\tilde{F}_\alpha, A) \hat{\cap}(\tilde{1}_1, A) = (\tilde{F}_\alpha, A)$ and $(\tilde{F}_\alpha, A) \hat{\cup}(\tilde{\emptyset}, A) = (\tilde{F}_\alpha, A)$

\( \forall (\tilde{F}_\alpha, A) \in \tilde{G}H. \)
Fuzzy Sets and Open Problems

- Fuzzy soft sets (FSS)
- Intuitionistic FSS
- Hesitant FSS
- Interval-valued FSS
- Interval-valued Intuitionistic FSS
- Interval-valued Hesitant FSS
- Interval-valued Intuitionistic Hesitant FSS
- Entropy of FSS
- Correlation of ...
- Similarity of ...
- Fuzzy N-Soft Sets (FNSS)
- FNSS
- Hesitant N-soft graphs,
- Hesitant N-soft hypergraphs,
- Hesitant Pythagorean fuzzy graphs
Intuitionistic fuzzy recommender systems: An effective tool for medical diagnosis

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Abstract

Medical diagnosis has been being considered as one of the important processes in clinical medicine that determines acquired diseases from some given symptoms. Enhancing the accuracy of diagnosis is the centralized focuses of researchers involving the uses of computerized techniques such as intuitionistic fuzzy sets (IFS) and recommender systems (RS). Based upon the observation that medical data are often imprecise, incomplete and vague so that using the standalone IFS and RS methods may not improve the accuracy of diagnosis, in this paper we consider the integration of IFS and RS into the proposed methodology and present a novel intuitionistic fuzzy recommender systems (IFRS) including: (i) new definitions of single-criterion and multi-criteria IFRS; (ii) new definitions of intuitionistic fuzzy matrix (IFM) and intuitionistic fuzzy composition matrix (IFCM); (iii) proposing intuitionistic fuzzy similarity matrix (IFSM), intuitionistic fuzzy similarity degree (IFSD) and the formulas to predict values on the basis of IFSD; (iv) a novel intuitionistic fuzzy collaborative filtering method so-called IFCF to predict the possible diseases. Experimental results reveal that IFCF obtains better accuracy than the standalone methods of IFS such as De et al., Szmidt and Kacprzyk, Samuel and Balamurugan and RS, e.g. Davis et al. and Hassan and Syed.

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A similarity measure of intuitionistic fuzzy soft sets and its application in medical diagnosis

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Similarity measure

\textbf{ABSTRACT}

In this paper, a new similarity measure and a weighted similarity measure on intuitionistic fuzzy soft sets (IFSSs) are proposed and some of their basic properties are discussed. Using the proposed similarity measure, a relation (=\textsuperscript{w}) between two IFSSs are defined and it is found that the defined relation is not an equivalence relation. Further, the effectiveness of the proposed similarity measure is demonstrated in a numerical example with the help of measure of performance and measure of error. Moreover, medical diagnosis problems have been exhibited through a hypothetical case study by using this proposed similarity measure. Finally, the proposed method is applied to 10 different medical data sets from UCI Machine Learning Repository datasets and its similarity measures are calculated. The corresponding performance measures, like accuracy, sensitivity, specificity, ROC curves, AUC values, and F-measures are obtained and it is compared with the existing methods. This shows that the proposed method exhibits more accuracy, sensitivity and enhanced F-measures than the existing methods.

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