Local Antimagic Coloring of Graphs

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Overview

1. Local Antimagic Vertex Coloring
2. Local Antimagic Total Vertex Coloring
3. Super Local Antimagic Total Vertex Coloring
4. Local Antimagic Edge Coloring
5. Conclusion

Local Antimagic Vertex Coloring

Introduction

Definition [Arumugam, Premalatha, Bača, Semaničová-Feňovčiková (2017)]

The local antimagic labeling on a graph G with $|V|$ vertices and $|E|$ edges is defined to be an assignment $f : E \to \{1, 2, \cdots, |E|\}$ so that the weights of any two adjacent vertices u and v are distinct, that is, $w(u) \neq w(v)$ where $w(u) = \Sigma_{e \in E(u)} f(e)$ and $E(u)$ is the set of edges incident to u.

- Any local antimagic labeling induces a proper vertex coloring of G where the vertex u is assigned the color $w(u)$.
- The local antimagic chromatic number, denoted by $\chi_{la}(G)$, is the minimum number of colors taken over all colorings induced by local antimagic labelings of G.
- For any graph G, $\chi_{la}(G) \geq \chi(G)$ and the difference $\chi_{la}(G) - \chi(G)$ can be arbitrarily large.

Example

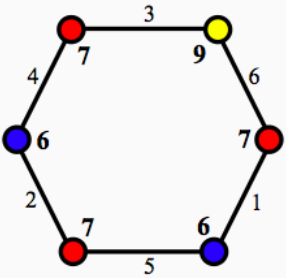

Figure 1: The local antimagic vertex coloring of C_6 with $\chi_{la}(C_6) = 3$

Cycle

Theorem [Arumugam, Premalatha, Bača, Semaničová-Feňovčiková (2017)]

For the **cycle** C_n on $n \geq 3$ vertices, $\chi_{la}(C_n) = 3$

Proof.

- Label the edges of C_n using the following formula.

$$f(v_i v_{i+1}) = \begin{cases} n - \frac{i-1}{2} & \text{if } i \text{ is odd} \\ \frac{i}{2} & \text{if } i \text{ is even} \end{cases}$$

- The weight of vertices are

$$w(v_i) = \begin{cases} n & \text{if } i \text{ is odd, } i \neq 1 \\ n+1 & \text{if } i \text{ is even} \\ 2n - \lfloor \frac{n}{2} \rfloor & \text{if } i = 1 \end{cases}$$

- Thus $\chi_{la}(C_n) \leq 3$.

Proof (cont.)

- Suppose n is even and there exists f that induces a 2-coloring of C_n.
- Let x be the color of v_i if i is odd and y if i is even.
- Then $\frac{n}{2}(x+y) = 2[\frac{n(n+1)}{2}]$ and hence $x+y = 2n+2$.
- If $f(v_i v_{i+1}) = n$, then $w(v_i)$ and $w(v_{i+1})$ are at least $n+1$ and $n+2$.
- Thus $x+y \geq 2n+3$, which is a contradiction.
- So $\chi_{la}(C_n) \geq 3$ if n is even.
- Further, $\chi(C_n) = 3$ if n is odd and hence $\chi_{la}(C_n) \geq 3$.
- Therefore $\chi_{la}(C_n) = 3$.

Tree

Theorem [Arumugam, Premalatha, Bača, Semaničová-Feňovčiková (2017)]

For any tree T with l leaves, $\chi_{la}(T) \geq l + 1$.

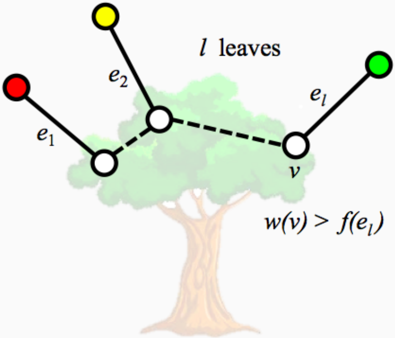

Figure 2: Tree with l leaves

Path

Theorem [Arumugam, Premalatha, Bača, Semaničová-Feňovčiková (2017)]

For path P_n on $n \geq 3$ vertices, $\chi_{la}(P_n) = 3$.

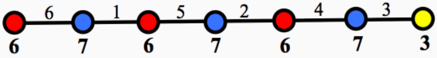

Figure 3: The local antimagic vertex coloring of P_7 with $\chi_{la}(P_7) = 3$

Friendship graph

Theorem [Arumugam, Premalatha, Bača, Semaničová-Feňovčiková (2017)]

For friendship graph F_n with $n \geq 2$, $\chi_{la}(F_n) = 3$.

Theorem [Arumugam, Premalatha, Bača, Semaničová-Feňovčiková (2017)]

For friendship graph F_n with $n \geq 2$ by removing an edge e, $\chi_{la}(F_n - \{e\}) = 3$.

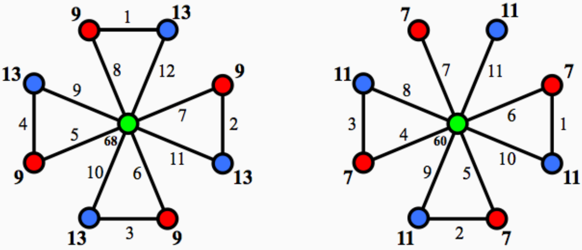

Figure 4: The local antimagic vertex coloring of F_4 and $F_4 - \{e\}$

Complete bipartite graph

Theorem **[Arumugam, Premalatha, Bača, Semaničová-Feňovčiková (2017)]**

For complete bipartite graph $K_{m,n}$ with $m, n \geq 2$, $\chi_{la}(K_{m,n}) = 2$ if and only if $m \equiv n \pmod 2$.

Theorem **[Arumugam, Premalatha, Bača, Semaničová-Feňovčiková (2017)]**

For complete bipartite graph $K_{2,n}$ with $n \geq 2$, $\chi_{la}(K_{2,n}) = 2$ for even $n \geq 2$ and $\chi_{la}(K_{2,n}) = 3$ for odd $n \geq 3$ or $n = 2$.

Graph L_n

Theorem [Arumugam, Premalatha, Bača, Semaničová-Feňovčiková (2017)]

Graph L_n for $n \geq 2$ that is obtained by inserting a vertex to each edge vv_i, $1 \leq i \leq n-1$, of the star, $\chi_{la}(L_n) = n+1$.

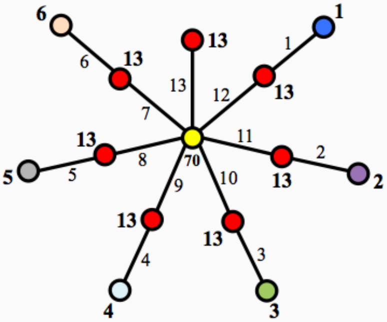

Figure 5: The local antimagic vertex coloring of L_7 with $\chi_{la}(L_7) = 8$

Wheel

Theorem [Arumugam, Premalatha, Bača, Semaničová-Feňovčiková (2017)]

Wheel W_n of order $n+1$ for $n \geq 3$, $\chi_{la}(W_n) = 4$ if $n \equiv 1, 3 \pmod 4$, $\chi_{la}(W_n) = 3$ if $n \equiv 2 \pmod 4$, and $3 \leq \chi_{la}(W_n) \leq 5$ if $n \equiv 0 \pmod 4$.

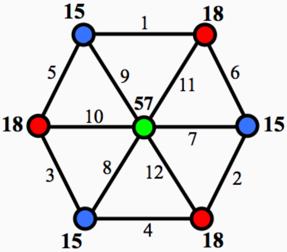

Figure 6: The local antimagic vertex coloring of W_6 with $\chi_{la}(W_6) = 3$

Fan

Corollary

Fan f_n of order $n+1$ for $n \geq 6$ and $n \equiv 2 \pmod 4$, $\chi_{la}(f_n) = 3$.

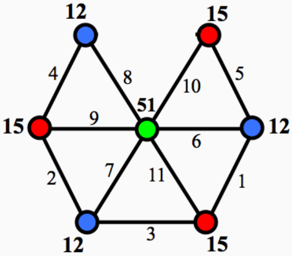

Figure 7: The local antimagic vertex coloring of f_6 with $\chi_{la}(f_6) = 3$

Graph resulting from operation

Theorem **[Arumugam, Premalatha, Bača, Semaničová-Feňovčiková (2017)]**

For the graph $H = G + \bar{K}_2$ where G is a graph of order $n \geq 4$, then

$$\chi_{la}(G) + 1 \leq \chi_{la}(H) \leq \begin{cases} \chi_{la}(G) + 1 & \text{for } n \text{ is even} \\ \chi_{la}(G) + 2 & \text{otherwise.} \end{cases}$$

Kite

Theorem [Nazula, S, Dafik (2018)]

For the kite $Kt_{n,m}$ with $n \geq 3$ and $m \geq 1$, $\chi_{la}(Kt_{n,m}) = 3$.

Proof.

- Label the edges of $Kt_{n,m}$ using the following formula.

$$f(u_i u_{i+1}) = \begin{cases} \frac{m+i}{2} & \text{if } m+i \text{ is even} \\ \frac{m+3-i}{2} + \lfloor \frac{n}{2} \rfloor + \lfloor \frac{n-1}{2} \rfloor & \text{if } m+i \text{ is odd} \end{cases}$$

$$f(u_n u_1) = \begin{cases} \lfloor \frac{n}{2} \rfloor + \frac{m+1}{2} & \text{if } m \text{ is odd} \\ \lfloor \frac{n+1}{2} \rfloor + \frac{m}{2} & \text{if } m \text{ is even} \end{cases}$$

$$f(u_1 v_1) = \begin{cases} \frac{m}{2} & \text{if } m \text{ is even} \\ \frac{m+1}{2} + n & \text{if } m \text{ is odd} \end{cases}$$

$$f(v_j v_{j+1}) = \begin{cases} \frac{m-j}{2} & \text{if } m+j \text{ is even} \\ \frac{m+j+1}{2} + n & \text{if } m+j \text{ is odd} \end{cases}$$

Proof (cont.)

- The weight of vertices are

$$w(u_i) = \begin{cases} \frac{3m}{2} + 2\lfloor\frac{n+1}{2}\rfloor + \lfloor\frac{n}{2}\rfloor & \text{if } m \text{ is even and } i = 1 \\ n + m + 1 & \text{if } m + i \text{ is even} \\ n + m & \text{if } m + i \text{ is odd} \\ \frac{3m+3}{2} + \lfloor\frac{n}{2}\rfloor + n & \text{if } m \text{ is odd and } i = 1 \end{cases}$$

$$w(v_i) = \begin{cases} n + m & \text{if } m + j \text{ is even} \\ n + m + 1 & \text{if } m + j \text{ is odd} \end{cases}$$

- Thus $\chi_{la}(Kt_{n,m}) \leq 3$.
- The lower bound uses $\chi_{la}(C_n)$ due to Arumugam *et al.*
- Since $\chi_{la}(C_n) = 3$ and the kite $Kt_{n,m}$ contains contains a subgraph that is isomorphic to the C_n, then $\chi_{la}(Kt_{n,m}) \geq 3$.
- Therefore $\chi_{la}(Kt_{n,m}) = 3$.

Kite (cont.)

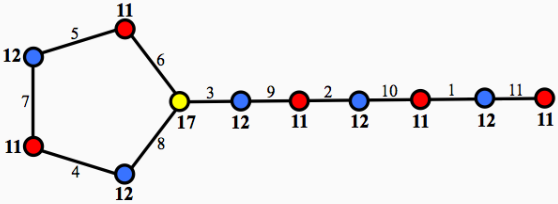

Figure 8: The local antimagic vertex coloring of $Kt_{5,6}$ with $\chi_{la}(Kt_{5,6}) = 3$

Cycle with two neighbour pendants

Theorem [Nazula, S, Dafik (2018)]

For cycle with two neighbour pendants Cp_n with $n \geq 3$, $\chi_{la}(Cp_n) = 4$.

Proof.

- Label the edges of Cp_n using the following formula.

$$
\begin{aligned}
f(u_i u_{i+1}) &= \begin{cases} \frac{i+1}{2} & \text{if } i \text{ is odd} \\ n+1-\frac{i}{2} & \text{if } i \text{ is even} \end{cases} \\
f(u_n u_1) &= \lceil \frac{n+1}{2} \rceil \\
f(u_i v_i) &= n+i \qquad \text{if } i = 1,2
\end{aligned}
$$

- The weight of vertices are

$$
\begin{aligned}
w(u_i) &= \begin{cases} \lfloor \frac{3n+6}{2} \rfloor & \text{if } i = 1 \\ 2n+3 & \text{if } i = 2 \\ n+2 & \text{if } i \text{ is odd and } i \geq 3 \\ n+1 & \text{if } i \text{ is even and } i \geq 4 \end{cases} \\
w(v_i) &= n+i \qquad \text{if } i = 1,2
\end{aligned}
$$

Cycle with two neighbour pendants (cont.)

Proof (cont.)

- Thus $\chi_{la}(Cp_n) \leq 4$.
- To show the lower bound, we suppose that $f(u_1v_1) = m_1$ and $f(u_2v_2) = m_2$.
- Then $w(v_1) = m_1$, $w(v_2) = m_2$, $w(u_1) > m_1$ and $w(u_2) > m_2$
- Clearly, $w(v_1) \neq w(v_2)$.
- As u_1 is neighbour of u_2, then $w(u_1) \neq w(u_2)$.
- This implies that $\chi_{la}(Cp_n) \geq 4$.
- We conclude that $\chi_{la}(Cp_n) = 4$.

Cycle with two neighbour pendants (cont.)

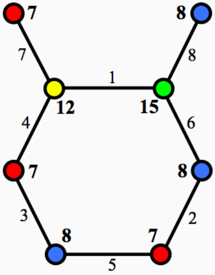

Figure 9: The local antimagic vertex coloring of Cp_6 with $\chi_{la}(Cp_6) = 4$

Corona Product of Graphs

Theorems [Arumugam, Lee, Premalatha, Wang (2018+)]

- $\chi_{la}(P_n \odot K_1) = n + 2$ for $n \geq 4$.
- $\chi_{la}(P_n \odot \bar{K}_m) = mn + 2$ for $m \geq 2$ and $n \geq 2$.
- $\chi_{la}(C_n \odot K_1) = n + 2$ for $n \geq 4$.
- $\chi_{la}(C_n \odot \bar{K}_2) = 2n + 2$ for $n \geq 4$.
- $\chi_{la}(C_3 \odot \bar{K}_2) = 3m + 3$ for $m \geq 2$.
- $\chi_{la}(C_n \odot \bar{K}_m) = mn + 3$ for odd $n \geq 5$ except finitely many m?s.
- $\chi_{la}(K_n \odot K_1) = 2n - 1$ for $n \geq 3$.
- $\chi_{la}(K_n \odot \bar{K}_m) = mn + n$ for $m \geq 2$ and $n \geq 3$.

Local Antimagic Total Vertex Coloring

Local Antimagic Total Vertex Coloring

Definition **[Putri, Dafik, Agustin, Alfarisi (2018)]**

The local vertex antimagic total labeling on a graph G with $|V|$ vertices and $|E|$ edges is defined to be an assignment $f : V \cup E \rightarrow \{1, 2, \cdots, |V| + |E|\}$ so that the weights of any two adjacent vertices u and v are distinct, that is, $w(u) \neq w(v)$ where $w(u) = f(u) + \Sigma_{e \in E(u)} f(e)$ and $E(u)$ is the set of edges incident to u.

- Any local vertex antimagic total labeling induces a proper vertex coloring of G where the vertex u is assigned the color $w(u)$.
- The local antimagic total chromatic number, denoted by $\chi_{lat}(G)$, is the minimum number of colors taken over all colorings induced by local vertex antimagic total labelings of G.
- For any graph G, $\chi_{lat}(G) \geq \chi(G)$.

Example

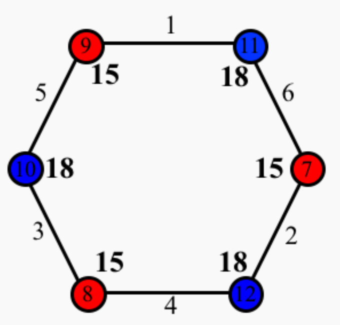

Figure 10: The local antimagic total vertex coloring of C_6 with $\chi_{lat}(C_6) = 2$

Particular class of trees

Putri, Dafik, Agustin, Alfarisi (2018)

- For star S_n with $n \geq 2$, $\chi_{lat}(S_n) = 2$.
- For double star $S_{n,m}$ with $n \geq 2$ and $m \geq 2$, $\chi_{lat}(S_{n,m}) \leq 3$.
- For banana tree $B_{m,n}$ with $n \geq 3$ and $m \geq 3$,

$$\chi_{lat}(B_{m,n}) \leq \begin{cases} 4, & \text{if } n \text{ odd, } m \text{ odd} \\ 5, & \text{if } n \text{ odd, } m \text{ even} \\ 6, & \text{if } n \text{ even, } m \text{ even} \end{cases}$$

Wheel

Theorem [S, Dafik, Hasan (2018)]

For wheel W_n of order $n+1$,

$$\chi_{lat}(W_n) = \begin{cases} 3, & n \equiv 0 \pmod 2 \\ 4, & n \equiv 1 \pmod 2). \end{cases}$$

Proof.

- Label all vertices and edges of W_n using the following formula.

$$\begin{aligned}
f(x_i x_{i+1}) &= i, && \text{for } 1 \le i \le n-1 \\
f(x_n x_1) &= n \\
f(cx_i) &= 2n+1-i, && \text{for } 1 \le i \le n \\
f(x_i) &= \begin{cases} 2n+2, & \text{for } i = 1 \\ 3n, & \text{for } i = 2,\ n \text{ odd} \\ 3n-i+2-(-1)^{i+n}, & \text{untuk } 2 \le i \le n \end{cases} \\
f(c) &= 3n+1.
\end{aligned}$$

Proof (cont.).

- The weight of the vertices are

$$w(x_i) = \begin{cases} 5n+1, & \text{for } (i+n) \text{ even} \\ 5n+2, & \text{for } i=2,\ n \text{ odd} \\ 5n+3, & \text{for } (i+n) \text{ odd} \quad \text{or } i=1, n \text{ odd} \end{cases}$$

$$w(c) = \frac{n(3n+7)}{2} + 1.$$

- Thus $\chi_{lat}(W_n) \leq 3$ for n even and $\chi_{lat}(W_n) \leq 4$ for n odd.
- Since $\chi_{lat}(W_n) \geq \chi(W_n) = 3$ for n even and $\chi_{lat}(W_n) \geq \chi(W_n) = 4$ for n odd.
- Therefore, $\chi_{lat}(W_n) = 3$ for n even and $\chi_{lat}(W_n) = 4$ for n odd.

Wheel (cont.)

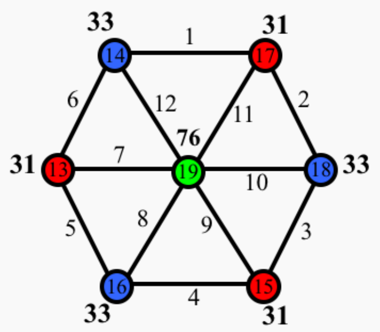

Figure 11: The local antimagic total vertex coloring of W_6 with $\chi_{lat}(W_6) = 3$

Other wheel related graphs

Theorems **[S, Dafik, Hasan (2018)]**

- For fan F_n of order $n \geq 3$, $\chi_{lat}(f_n) = 3$
- For friendship graph f_n of order $2n + 1$ with $n \geq 2$, $\chi_{lat}(f_n) = 3$

Broom

Theorem [Nikmah, S, Hobri (2018)]

For broom $B_{n,m}$ with $m \geq 3$ and $n - m \geq 2$, $\chi_{la}(B_{n,m}) = n - m + 2$

Theorem [Nikmah, S, Hobri (2018)]

For double broom $B_{n,m}$ with $m \geq 3$ and $n - m \geq 2$,

1. $\chi_{lat}(B_{n,m}) \leq 3$, and
2. $\chi_{lat}(B_{n,m}) = 3$, if $n > \dfrac{7 + 2m + \sqrt{48m - 23}}{2}$.

Prism and Möbius ladder

Theorem **[Hasan, S, Dafik (2018)]**

For prism D_n of order $2n$ with $n \geq 3$,

$$\chi_{lat}(D_n) = \begin{cases} 2, & n \equiv 0 (\text{mod } 2) \\ 3, & n \equiv 1 (\text{mod } 2). \end{cases}$$

Theorem **[Hasan, S, Dafik (2018)]**

For Möbius ladder M_{2n} of order $2n$ with $n \geq 3$,

$$\chi_{lat}(M_{2n}) = \begin{cases} 3, & n \equiv 0 (\text{mod } 2) \\ 2, & n \equiv 1 (\text{mod } 2). \end{cases}$$

Super Local Antimagic Total Vertex Coloring

Super Local Antimagic Total Vertex Coloring

Definition

The super local vertex antimagic total labeling on a graph G with $|V|$ vertices and $|E|$ edges is defined as local vertex antimagic total labeling where the vertices of G receive the smallest labels, that is, $1, 2, ..., |V|$.

- Any super local vertex antimagic total labeling induces a proper vertex coloring of G where the vertex u is assigned the color $w(u)$.
- The super local antimagic total chromatic number, denoted by $\chi_{slat}(G)$, is the minimum number of colors taken over all colorings induced by super local vertex antimagic total labelings of G.
- For any graph G, $\chi_{slat}(G) \geq \chi_{lat}(G) \geq \chi(G)$.

Example

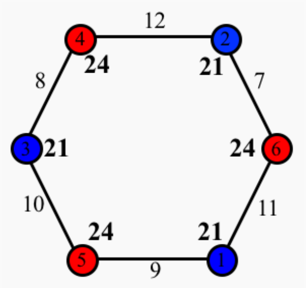

Figure 12: The super local antimagic total vertex coloring of C_6 with $\chi_{slat}(C_6) = 2$

Some results

Theorem **[S, Dafik, Hasan (2018+)]**

For cycle C_n of order $n \geq 3$,

$$\chi_{las}(C_n) = \begin{cases} 3, & \text{if } n \text{ is odd} \\ 2, & \text{if } n \text{ is even.} \end{cases}$$

Theorem **[S, Dafik, Hasan (2018+)]**

For path P_n of order $n \geq 4$, $3 \leq \chi_{slat}(P_n) \leq 4$,

Gear and Generalised Wheel

Theorem **[S, Dafik, Hasan (2018+)]**

For gear $J_{n,1}$ of order $2n + 1$ and odd $n \geq 5$, $\chi_{slat}(J_{n,1}) = 3$.

Theorem **[S, Dafik, Hasan (2018+)]**

For generalized wheel W_m^n of order $mn + 1$ where $m \geq 1$ and $n \geq 3$,

$$\chi_{slat}(W_m^n) = \begin{cases} 3, & n \text{ even} \\ 4, & n \text{ odd} \end{cases}$$

Local Antimagic Edge Coloring

Local Antimagic Edge Coloring

Definition [Agustin, Hasan, Dafik, Alfarisi, Prihandini (2017)]

The local edge antimagic labeling on a graph G with $|V|$ vertices and $|E|$ edges is defined to be an assignment $f : V \to \{1, 2, \cdots, |V|\}$ so that the weights of any adjacent edges, that is $\{w(uv) : w(uv) = f(u) + f(v), uv \in E\}$, are distinct.

- Any local edge antimagic labeling induces a proper edge coloring of G where the edge uv is assigned the color $w(uv)$.
- The local antimagic edge chromatic number, denoted by $\chi_{lea}(G)$, is the minimum number of colors taken over all colorings induced by local edge antimagic labelings of G.
- For any graph G, $\chi_{lae}(G) \geq \chi'(G)$.

Theorem [Agustin, Hasan, Dafik, Alfarisi, Prihandini (2017)]

For the **cycle** C_n on $n \geq 3$ vertices, $\chi_{lea}(C_n) = \chi_{la}(C_n) = 3$

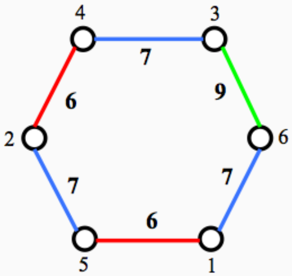

Figure 13: The local antimagic edge coloring of C_6 with $\chi_{lea}(C_6) = 3$

Path

Theorem [Agustin, Hasan, Dafik, Alfarisi, Prihandini (2017)]

For the **path** P_n on $n \geq 2$ vertices, $\chi_{lea}(P_n) = 2$.

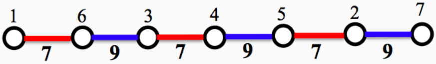

Figure 14: The local antimagic edge coloring of P_7 with $\chi_{lea}(P_7) = 2$

Complete Graph

Theorem **[Agustin, Hasan, Dafik, Alfarisi, Prihandini (2017)]**

For the **complete graph** K_n on $n \geq 3$ vertices, $\chi_{lea}(K_n) = 2n - 3$

Wheel related graphs [Agustin, Hasan, Dafik, Alfarisi, Prihandini (2017)]

- Wheel W_n for $n \geq 3$, $\chi_{lea}(f_n) = n + 2$
- Fan graph f_n for $n \geq 3$, $\chi_{lea}(f_n) = n + 1$
- Gear graph $J_{n,1}$ for $n \geq 3$, $\chi_{lea}(J_{n,1}) = n + 2$
- Helm graph H_n for $n \geq 3$, $\chi_{lea}(H_n) = n + 3$
- Flower graph Fl_n for $n \geq 3$, $\chi_{lea}(Fl_n) = 2n + 1$
- Sun flower graph SFl_n for $n \geq 3$, $\chi_{lea}(SFl_n) = 2n - 1$

Corona Product of Graphs

Theorem **[Agustin, Hasan, Dafik, Alfarisi, Prihandini (2017)]**

For corona product of cycle C_n and m copy of K_1 where $n \geq 3$ and $m \geq 1$, $\chi_{lea}(C_n \odot mK_1) = m + 3$,

Theorem **[Agustin, Hasan, Dafik, Alfarisi, Prihandini (2017)]**

For corona product of any graph G of order $n \geq 3$ and m copy of K_1 where $m \geq 1$, $\chi_{lea}(G \odot mK_1) = \chi_{lea}(G) + m$,

Local Antimagic Total Edge Coloring

Definition [I.H. Agustin, Dafik, M. Hasan, 2017]

The local edge antimagic total labeling on a graph G with $|V|$ vertices and $|E|$ edges is defined to be an assignment $f : V \cup E \to \{1, 2, \cdots, |V| + |E|\}$ so that the weights of any adjacent edges, that is $\{w(uv) : w(uv) = f(u) + f(uv) + f(v), uv \in E\}$, are distinct.

- Any local edge antimagic total labeling induces a proper edge coloring of G where the edge uv is assigned the color $w(uv)$.
- The local antimagic total edge chromatic number, denoted by $\chi_{leat}(G)$, is the minimum number of colors taken over all colorings induced by local edge antimagic total labelings of G.
- For any graph G, $\chi_{leat}(G) \geq \chi'(G)$.
- If the vertices of G receive the smallest labels, that is, $1, 2, ..., |V|$, then the such labeling is called super local edge antimagic total labeling.
- Any super local edge antimagic total labeling also induces a proper edge coloring of G.

Conclusion

Summary of local antimagic labeling

Labeling	Label		Weight		Local Antimagic Chromatic Number
	Vertex	Edge	Vertex	Edge	
Local Antimagic		x	x		
Local Antimagic Total	x	x	x		$\chi_{lat} \geq \chi_{slat} \geq \chi_{la} \geq \chi$
Super Local Antimagic Total	x (min)	x	x		
Local Edge Antimagic		x		x	
Local Edge Antimagic Total	x	x		x	$\chi_{leat} \geq \chi_{sleat} \geq \chi_{lea} \geq \chi'$
Super Local Edge Antimagic Total	x (min)	x		x	

Summary of known results on local antimagic labeling

Graphs	χ_{la}	χ_{lat}	χ_{slat}	χ_{lea}	χ_{leat}	χ_{sleat}
Cycle C_n, $n \geq 3$	3	3, if n is odd 2, if n is even	3, if n is odd 2, if n is even	3	3, if n is odd 2, if n is even	3, if n is odd 2, if n is even
Path P_n, $n \geq 3$	3	3	$3 \leq \chi \leq 4$	2	2, if n is odd 3, if n is even	?
Wheel W_n, $n \geq 3$	4, if $n \equiv 1,3 \pmod 4$ 3, if $n \equiv 2 \pmod 4$ $3 \leq \chi \leq 5$ if $n \equiv 0 \pmod 4$	4, if n is odd 3, if n is even	4, if n is odd 3, if n is even	$n+2$	$n+1$	?
Fan F_n, $n \geq 3$	3	3	?	$n+1$	$n+1$	?
Friendship f_n, $n \geq 2$	3	3	3	$2n+1$	$2n+1$	?
Complete Bipartite Graph $K_{m,n}$, $m,n \geq 2$	$2 \leftrightarrow m \equiv n \pmod 2$ 2, if m=2, n even 3, if m=2, n odd	2	2	?	?	?
Complete Graph K_n, $n \geq 3$	n	n	n	$2n - 3$	?	?
Star $K_{1,n-1}$, $n \geq 3$	n	2	2	n - 1	?	?
Tree T with l leaves	$\geq l+1$	3, if l=n-2	3, if l=n-2	?	?	?
Kite $Kt_{n,m}$, $n \geq 3$, $m \geq 1$	3	?	?	?	3	3
Cycle with 2 pendants	4	?	?	?	?	?
Prism D_n, $n \geq 3$	?	3, if n is odd 2, if n is even	?	5	?	?
Mobius Ladder M_{2n}, $n \geq 3$	?	2, if n is odd 3, if n is even	?	?	?	?
Gear $J_{n,1}$ $n \geq 5$	?	3	3	?	$n+1$	?

Note: refer to previous slides for the authors

Other graphs???

Open Problems

Open Problem 1

Determine the (super) local antimagic (total) (edge) chromatic number of other class of graphs G.

Open Problem 2 [Arumugam, Premalatha, Bača, Semaničová-Feňovčiková (2017)]

Characterize the class of graphs G for which $\chi_{la}(G) = \chi(G)$.

Open Problem 3

Characterize the class of graphs G for which $\chi_{lat}(G) = \chi(G)$, $\chi_{lea}(G) = \chi'(G)$, or $\chi_{leat}(G) = \chi'(G)$ and $\chi_{la}(G) = \chi_{lat}(G) = \chi_{slat}(G)$ or $\chi_{lea}(G) = \chi_{leat}(G) = \chi_{sleat}(G)$.

Open Problem 4

How about local irregular labeling vertex or edge coloring?

THANK YOU