SEMESTER STUDY PLAN
ADVANCED LINEAR ALGEBRA
(COMPULSORY COURSE)

DEPARTMENT OF MATHEMATICS AND DATA SCIENCE
FACULTY OF MATHEMATICS AND NATURAL SCIENCES
UNIVERSITAS ANDALAS

2024

<table>
<tr><td colspan="2"></td><td colspan="6">SEMESTER STUDY PLAN (SSP)
MASTER PROGRAM OF MATHEMATICS
FACULTY OF MATHEMATICS AND NATURAL SCIENCES
UNIVERSITAS ANDALAS</td></tr>
<tr><th colspan="2">Course Name</th><th>Course Code</th><th colspan="2">URL I-Learn</th><th>Credits</th><th>Semester</th><th>Compilation Date</th></tr>
<tr><td colspan="2">Advanced Linear Algebra</td><td>MAT81111</td><td colspan="2">https://sci.ilearn.unand.ac.id</td><td>3</td><td>1</td><td>15 May 2024</td></tr>
<tr><th colspan="2" rowspan="2">Person In Charge</th><th colspan="2">Study Plan Creator</th><th colspan="2">Head of Research Group</th><th colspan="2">Head of Study Program</th></tr>
<tr><td colspan="2">Prof. Dr. Admi Nazra
Dr. Yanita</td><td colspan="2">Nova Noliza Bakar, M.Si</td><td colspan="2">Prof. Dr. Ferra Yanuar</td></tr>
<tr><td rowspan="6">Intended Learning Outcomes (ILO) and Performance Indicator (PI)</td><th colspan="7">Intended Learning Outcomes</th></tr>
<tr><td>ILO-2</td><td colspan="6">Mastering mathematical concepts and applications (real analysis, advanced linear algebra, and statistics) in solving complex mathematical problems
PI-1: An ability explain mathematical concepts (Real Analysis, Advanced Linear Algebra, and Statistics).
PI-2: An ability to identify complex mathematical problems.
PI-3: An ability to solve complex mathematical problems.</td></tr>
<tr><td>ILO-3</td><td colspan="6">Comprehensive mastery of one or several theories for development in the fields of analysis, algebra, applied mathematics, statistics and combinatorial mathematics.
PI-1: An ability to identify theories used in related mathematical problems.
PI-2: An ability to apply theories for advancement in related fields (advanced theory).
PI-3: An ability to use advanced theory to solve related mathematical problems.</td></tr>
<tr><th colspan="7">Course Learning Outcomes</th></tr>
<tr><td>1</td><td colspan="6">An ability to understand/solve problems and properties in vector space (ILO-2: P1-1, PI-2, PI-3, ILO-3: P1-1, PI-2, PI-3)</td></tr>
</table>

<table>
<tr><td rowspan="3"></td><td>2</td><td colspan="2">An ability to understand/solve problems and properties of eigenvalues and eigenvectors (ILO-2: P1-1, PI-2, PI-3, ILO-3: P1-1, PI-2, PI-3)</td></tr>
<tr><td>3</td><td colspan="2">An ability to understand/solve problems and properties of linear transformations (ILO-2: P1-1, PI-2, PI-3, ILO-3: P1-1, PI-2, PI-3)</td></tr>
<tr><td>4</td><td colspan="2">An ability to solve problems and properties in quotidian spaces and isomorphism theorems (ILO-2: P1-1, PI-2, PI-3, ILO-3: P1-1, PI-2, PI-3)</td></tr>
<tr><td>Brief Description</td><td colspan="3">In this course, mathematical concepts will be discussed in the form of definitions and mathematical properties in the form of lemmas and theorems related to Linear Algebra, which include: vector spaces and subspaces, the basis of a vector space, eigenvalues and eigenvectors, diagonalization, and transformations. linear and representational matrices, quotidian spaces and isomorphism theorem.

The learning method in this course is face to face</td></tr>
<tr><td>Course Materials</td><td colspan="3">1. Vector Space
2. Eigenvalues and Eigenvectors
3. Linear Transformation</td></tr>
<tr><td>References</td><td colspan="3">Main:
1. Steven Roman, Advanced Linear Algebra, Springer Science+Business. 2008
Additional:
2. Hugo, J. Woerdeman, Advanced Linear Algebra, 2nd eds. Taylor & Francis Group , New York, 2016.
3. Bruce, N. Cooperstein, Advanced Linear Algebra, Taylor & Francis Group , New York, 2015</td></tr>
<tr><td rowspan="2">Learning Media</td><td colspan="2">Software:</td><td>Hardware:</td></tr>
<tr><td colspan="2">• LMS Unand (http://fmipa.ilearn.unand.ac.id/)
• Zoom meeting
• Whatsapp</td><td>• Computer/Laptop
• Smartphone</td></tr>
</table>

Team Teaching	1. Prof. Dr. Admi Nazra 2. Dr. Yanita
Assessment	Homework, Quizzes, Mid-Term exam, Final exam
Required courses	-
Academic Norms	https://akademik.unand.ac.id/images/2022-03-30%20Peraturan%20Rektor%20Nomor%207%20Tahun%202022%20Penyelenggaraan%20Pendidikan-khusus%20Bab%20II.pdf

Weekly Study Plan

Week / Meet (1)	Course Outcomes (2)	Indicator (3)	Assessment (4)	Activities/Forms of Learning [Time estimated]					Subject, references (10)	Weight (11)
				Synchronous*		Asynchronous**		Media (9)		
				Face to face Offline (5)	Face to face Online (6)	Individual (7)	Collaboration (8)			
1	An ability to understand /solve problems and properties in vector spaces	• The accuracy in proving a nonempty set is a vector space • Accuracy in proving a subset is a subspace of vector space. • Accuracy in proving	• Non Test: Homework 1: 4% • Mid-term Exam: 5%	Teaching and discussion: • introduction of RPS • Explanation of Learning Material • Task Description • Assessment Explained	Teaching and discussion: • introduction of RPS • Explanation of Learning Material • Task Description • Assessment Explained	• Students read and study learning materials • Students do assignments independently		PPT I learn (LMS Unand) (Certain conditions: Zoom meeting, WA group, learning video)	• Lecture Contract (SSP) • **Theory Review** - Group theory - Field Ring Theory →Function - Elementary Line	**9%**

		Theorem Suppose the vector space over the field . A nonempty set is a subspace of if and only if$VFS \subset VV$ The set is closed to the summation, i.e. if then $.Ss_1, s_2 \in Ss_1 + s_2 \in S$ The set is closed to the scalar multiplication , i.e. if and then $.Sr \in Fs \in Srs \in S$		[1 x 3 x 50 menit]	[1 x 3 x 50 menit] (Certain conditions: Total *blended learning* : 50%)				Operations - Determinants of matrices - Inverse Matrix • **Main Material** - Vector space - Subspace of a vector space [1]	
2	An ability to understand /solve problems and properties in vector spaces	• Accuracy in proving **Proposition** Suppose and is a vector subspace of , then STV is a subspace of $.S \cap TV$ is a subspace of	Mid-term Exam: 5%	Teaching and discussions: • Explanation of Learning Material • Task Description	Teaching and discussions: • Explanation of Learning Material • Task Description	• Students read and study learning materials • Students do assignments independently		PPT I learn (LMS Unand) • (Certain conditions: Zoom	Lattice subspace [1]	5%

		.$S + T = \{s \in S, t \in T\} V$ ● Accuracy in proving **Proposition** If , then the subspace of , and the largest lower bound of and is $S, T \in S(V) S \cap T V S T glb\{S, T\} = S \cap T$ If , then is the subspace of and the smallest upper bound of and is $S, T \in S(V) S + T V S T lub\{S, T\} = S + T$		[1 x 3 x 50 minutes]	[1 x 3 x 50 minutes] (Certain conditions: Total *blended learning* : 50%)			meeting, WA group, learning video)		
3	An ability to understand /solve problems and properties in vector spaces	● The accuracy of proving one vector is a linear combination of one or more other vectors.	Non-Test Homework 2: 4% Mid-term Exam 3%	Teaching and discussions: ● Explanation of Learning Material ● Task Description	● Teaching and discussions: ● Explanation of Learning Material ● Task Description	● Students read and study learning materials ● Students do assignments independently		PPT I learn (LMS Unand) ● (Certain conditions:	● Linear combination ● Builder set (span) ● Linear Free	**7%**

		• Accuracy in proving one vector or several vectors is a builder in a vector space • The accuracy of proving one or more vectors is linearly independent or linearly dependent • Accuracy in proving the nature of the relationship of the set of builders and linear free, that is, proving **Theorem** Suppose the subset is nonempty in . Then the following		[1 x 3 x 50 minutes]	[1 x 3 x 50 minutes] (Specific conditions: Total *blended learning* : 40%)			Zoom meeting, WA group, learning video)		

		statement is equivalent:$S \neq \{0\}V$ 1. is linearly free$S$ • Each nonzero vector is a single linear combination of vectors in .$v \in span(S)S$ • No vector in is a linear combination of other vectors in SS **Theorem** Suppose the vector space over the field and the set of vectors in . The following statement is equivalent: $VFSV$ S is linearly free and is a span of .SV								

		For each vector, there is a single set of vectors together with a single set of scalar finite$v \in V v_1, v_2, \dots, v_n$ $S r_1, r_2, \dots r_n \in F$ $v = r_1 v_1 + r_2 v_2 + \cdots + r_n v_n$ 3. Suppose , then the building set is minimal if any subset of is not a span .$V = span(S) S S V$ • 4. Suppose, then the linearly free set is maximum, if any super-true set of depends linearly.$V = span(S) S S$								

4	An ability to understand / solve problems and properties in vector spaces	The accuracy in proving a set of vectors in a vector space is the basis Accuracy in proving **Theorem** Suppose the vector space and . Suppose is a linearly free set in and is the set of builders in which contains . Then there is a base in with Particularly: $VV \neq \{0\} IVSVIBVI \subseteq B \subseteq S$. Any vector space, except the vector space has a base.$\{0\}$,	Mid-term Exam: 5%	Teaching and discussions: Explanation of Learning Material [1 x 3 x 50 minutes]	Teaching and discussions: Explanation of Learning Material [1 x 3 x 50 minutes] (Specific conditions: Total *blended learning* : 40%)	Students read and study learning materials		PPT I learn (LMS Unand) (Certain conditions: Zoom meeting, WA group, learning video)	Basis Dimension	**5%**

		Any linear free repertoire is loaded in the base$V$ Any builder set in , loads the base.V Accuracy in proving • **Suppose a** V vector space and assume that the vectors are linearly free, and the vectors span from . So. $v_1, v_2, \dots, v_n u$ m								
5	An ability to understand /solve problems and properties in	• Accuracy in proving that a vector space is a direct sum • Accuracy in proving	Non-Test Homework 3: 2% Mid-term Exam : 5%	Teaching and discussions: Explanation of Learning Material	Teaching and discussions: Explanation of Learning Material Task Description	• Students read and study learning materials • Students do assignments independently		PPT I learn (LMS Unand) •(Certain conditio	• Direct su on vector space • Row space and column space'	**8%**

	vector spaces	**Theorem** Suppose a subspace of a vector space . For , , the form . Then if and only if fulfilled: $U_1, U_2, \ldots, U_k V i \in N 1 \leq i \leq k W_i = \sum_{j \neq i} U_i V = U_1 \oplus U_2 \oplus \ldots \oplus U_k$ $V = U_1 + U_2 + \cdots + U_k$ $U_i \cap W_i = \{0\}$ for each i • Accuracy in proving **Theorem** Suppose and are subspaces of vector spaces . So STV $dim\, dim\, (S) + dim\, dim\, (T) = dim\, dim\, (S + T) + dim\, (S \cap T)$		Task Description [1 x 3 x 50 minutes]	[1 x 3 x 50 minutes] (Specific conditions: Total *blended learning* : 40%)			ns: Zoom meeting, WA group, learning video)	• Null space	

		Specifically, if is the complement of , then TS $dim\,dim\,(S) + dim\,dim\,(T) = dim\,dim\,(V)$ that is $dim\,dim\,(S \oplus T) = dim\,dim\,(S) + dim\,dim\,(T)$ • Accuracy in determining vector coordinates from those associated with ordered bases (by changing the vector sequence) vB • Accuracy in proving **Theorem** Suppose the base is ordered in								

		dimensioned . Suppose and . So $B = (v_1, v_2, \dots, v_n)V$ $Vr \in F$ $[\square + \square]_{\square} = [u]_B + [v]_B$ $[rv]_B = r[v]_B$ • Accuracy in determining the row space, column space and nul space of a matrix, along with their bases • Accuracy in determining the rank and nullity of a matrix								
6	An ability to understand / solve problems	• Accuracy in determining eigenvalue	Non-Test Homework 4 : 5%	Teaching and discussions: Explanation of Learning Material	Teaching and discussions: Explanation of Learning Material	• Students read and study learning materials		PPT I learn (LMS Unand)	• Characteristic polynomials • Eigen	**7%**

	and properties in eigenvalues and eigenvectors	s and eigenvectors in a matrix ● Accuracy in proving **Theorem** Suppose is an eigenvalue – an eigenvalue that differs from a matrix. Then the eigenvectors associated with each of these eigenvalues are linearly free, that is, if , then the set is linearly free. $\lambda_1, \lambda_2, \dots, \lambda_k$ $A \in M_n(F) v_i \in E_{\lambda_i} \{v_1, v_2, \dots, v_k\}$ Accuracy in determining the algebraic multiplicity and geometric	Mid-term Exam : 3%	Task Description [1 x 3 x 50 minutes]	Task Description [1 x 3 x 50 minutes] (Specific conditions: Total *blended learning* : 50%)	● Students do assignments independently		●(Certain conditions: Zoom meeting, WA group, learning video)	value than vector eigen	

		multiplicity of an eigenvalue								
7	An ability to understand / solve problems and properties in eigenvalues and eigenvectors	• Accuracy in determining the invertible matrix that diagonalizes the matrix PA • Accuracy in determining digonalized and non-diagonalizable matrices • Accuracy in proving **Theorem** If the matrix , then the following two statements are equivalent: $An \times n$ The matrix can be diagonalized A The matrix has an eigenvector	• Non Test: Homework 5 : 5% • Mid-term Exam: 4%	Teaching and discussions: Explanation of Learning Material Task Description [1 x 3 x 50 minutes]	Teaching and discussions: Explanation of Learning Material Task Description [1 x 3 x 50 minutes] (Specific conditions: Total *blended learning* : 50%)	• Students read and study learning materials • Students do assignments independently		PPT I learn (LMS Unand) • (Certain conditions: Zoom meeting, WA group, learning video)	Diagonalization	**9%**

		that is linearly freeAn ● Accuracy in proving **Theorem** Suppose , then a matrix can be realized if and only if the geometric multiplicity of each of its eigenvalues is equal to its algebraic multiplicity. $A \in M_n(F)A$ ● Accuracy in proving **Theorem** Suppose and suppose the roots are in linear roots (of degree 1).$A \in M_n(F)C_A(x)$ A matrix is diagonalized if and only if the sum of the								

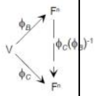
		geometric multiplicity of eigenvalues is An. If the geometric multiplicity of any eigenvalue of is equal to the algebraic multiplicity, it is diagonalized. A A If all the roots of are different (i.e. each of their algebraic multiplicity is 1), then they are diagonalized. $C_A(x)A$								
8	**Mid-Exam**									
9	An ability to understand /solve problems and	• Accuracy in proving a transformation is linear	• Non Test Homework 6 : 4%	Teaching and discussions: Explanation of Learning Material	Teaching and discussions: Explanation of Learning Material	• Students read and study learning materials		PPT I learn (LMS Unand)	• Linear transformation • Type of linear transforma	**8%**

	properties in linear transformations	• Accuracy in proving the properties of the himpunan linear transformation, i.e. accuracy in proving **Teorema** 1) *The set $\mathcal{L}(V,W)$ is a vector space under ordinary addition of functions and scalar multiplication of functions by elements of F.* 2) *If $\sigma \in \mathcal{L}(U,V)$ and $\tau \in \mathcal{L}(V,W)$, then the composition $\tau\sigma$ is in $\mathcal{L}(U,W)$.* 3) *If $\tau \in \mathcal{L}(V,W)$ is bijective then $\tau^{-1} \in \mathcal{L}(W,V)$.* 4) *The vector space $\mathcal{L}(V)$ is an algebra, where multiplication is composition of functions. The identity map $\iota \in \mathcal{L}(V)$ is the multiplicative identity and the zero map $0 \in \mathcal{L}(V)$ is the additive identity.* **Teorema** **Theorem 2.2** *Let V and W be vector spaces and let $\mathcal{B} = \{v_i \mid i \in I\}$ be a basis for V. Then we can define a linear transformation $\tau \in \mathcal{L}(V,W)$ by specifying the values of τv_i* arbitrarily *for all $v_i \in \mathcal{B}$ and extending τ to V by linearity, that is,* $\tau(a_1 v_1 + \cdots + a_n v_n) = a_1 \tau v_1 + \cdots + a_n \tau v_n$ *This process defines a unique linear transformation, that is, if $\tau, \sigma \in \mathcal{L}(V,W)$ satisfy $\tau v_i = \sigma v_i$ for all $v_i \in \mathcal{B}$ then $\tau = \sigma$.* • Accuracy in determining the kernel and image of a linear transformation. • Accuracy in	• Final Exam : 4%	Task Description [1 x 3 x 50 minutes]	Task Description [1 x 3 x 50 minutes] (Specific conditions: Total *blended learning* : 50%)	• Students do assignments independently		• (Certain conditions: Zoom meeting, WA group, learning video)	tion : endomorfisma, monomorfisma, epimorfisma, isomorfisma	

		proving properties in the type of linear transformation, namely **Theorem** **Theorem 2.3** *Let* $\tau \in \mathcal{L}(V, W)$*. Then* *1)* τ *is surjective if and only if* $\mathrm{im}(\tau) = W$ *2)* τ *is injective if and only if* $\ker(\tau) = \{0\}$									
10	An ability to understand/ solve problems and properties in linear transformations	• Accuracy in determining a linear transformation is bijective (isomorphism) • Accuracy in proving the properties of isomorphism, namely **Teorema**	Non Test Homework 7 : 4% Final Exam : 4%	Teaching and discussions: Explanation of Learning Material Task Description [1 x 3 x 50 minutes]	Teaching and discussions: Explanation of Learning Material Task Description [1 x 3 x 50 minutes] (Specific conditions: Total *blended learning* : 50%)	• Students read and study learning materials • Students do assignments independently		PPT I learn (LMS Unand) •(Certain conditions: Zoom meeting, WA group, learning video)	• Kernels and images • Kernel and image associations with linear transformation types	**8%**

		Theorem 2.4 *Let* $\tau \in \mathcal{L}(V, W)$ *be an isomorphism. Let* $S \subseteq V$*. Then* 1) S *spans* V *if and only if* τS *spans* W. 2) S *is linearly independent in* V *if and only if* τS *is linearly independent in* W. 3) S *is a basis for* V *if and only if* τS *is a basis for* W.□ **Theorem 2.5** *A linear transformation* $\tau \in \mathcal{L}(V, W)$ *is an isomorphism if and only if there is a basis* $\mathcal{B}$ *for* V *for which* $\tau\mathcal{B}$ *is a basis for* W*. In this case,* τ *maps any basis of* V *to a basis of* W.□ **Theorem 2.6** *Let* V *and* W *be vector spaces over* F*. Then* $V \approx W$ *if and only if* $\dim(V) = \dim(W)$.□ • Accuracy in determining the rank and nullity of a linear transformation • Accuracy in proving the nature of rank and nullity, namely **Theorem 2.8** *Let* $\tau \in \mathcal{L}(V, W)$. 1) *Any complement of* $\ker(\tau)$ *is isomorphic to* $\mathrm{im}(\tau)$ 2) **(The rank plus nullity theorem)** $\dim(\ker(\tau)) + \dim(\mathrm{im}(\tau)) = \dim(V)$ *or, in other notation,* $\mathrm{rk}(\tau) + \mathrm{null}(\tau) = \dim(V)$							
11	An ability to understand/ solve	• Accuracy in determining	Non Test Homework 8 : 4%	Teaching and discussions:	Teaching and discussions:	• Students read and study learning materials		PPT	Standard matrix	**8%**

	problems and properties in linear transformations	standard matrices based on definitions • Accuracy in proving properties on standard matrices • Accuracy in proving theorems **Theorem 2.10** 1) *If A is an $m \times n$ matrix over F then $\tau_A \in \mathcal{L}(F^n, F^m)$.* 2) *If $\tau \in \mathcal{L}(F^n, F^m)$ then $\tau = \tau_A$, where* $A = (\tau e_1 \mid \cdots \mid \tau e_n)$ *The matrix A is called the* **matrix** *of τ.* **Theorem 2.11** *Let A be an $m \times n$ matrix over F.* 1) $\tau_A: F^n \to F^m$ *is injective if and only if* $\text{rk}(A) = n$. 2) $\tau_A: F^n \to F^m$ *is surjective if and only if* $\text{rk}(A) = m$.	Final Exam : 4%	Explanation of Learning Material Task Description [1 x 3 x 50 minutes]	Explanation of Learning Material Task Description [1 x 3 x 50 minutes] (Specific conditions: Total *blended learning* : 50%)	• Students do assignments independently		I learn (LMS Unand) • (Certain conditions: Zoom meeting, WA group, learning video)		
12	An ability to understand / solve problems and properties in linear transformations	Accuracy in determining the matrix of related representations of a linear transformation related to each	Final exam: 6%	Teaching and discussions: Explanation of Learning Material [1 x 3 x 50 minutes]	Teaching and discussions: Explanation of Learning Material	Students read and study learning materials		PPT I learn (LMS Unand) • (Certain conditions: Zoom meeting,	• Transition matrix • Matriks similar	**6%**

		ordered base on domain vector space and codomain vector space  *Figure 2.1* the map that takes $[v]_{\mathcal{B}}$ to $[v]_{\mathcal{C}}$ is $\phi_{\mathcal{B},\mathcal{C}} = \phi_{\mathcal{C}}\phi_{\mathcal{B}}^{-1}$ and is called the **change of basis operator** (or **change of coordinates operator**). Since $\phi_{\mathcal{B},\mathcal{C}}$ is an operator on F^n, it has the form τ_A, where $A = (\phi_{\mathcal{B},\mathcal{C}}(e_1) \mid \cdots \mid \phi_{\mathcal{B},\mathcal{C}}(e_n)) = (\phi_{\mathcal{C}}\phi_{\mathcal{B}}^{-1}([b_1]_{\mathcal{B}}) \mid \cdots \mid \phi_{\mathcal{C}}\phi_{\mathcal{B}}^{-1}([b_n]_{\mathcal{B}})) = ([b_1]_{\mathcal{C}} \mid \cdots \mid [b_n]_{\mathcal{C}})$ We denote A by $M_{\mathcal{B},\mathcal{C}}$ and call it the **change of basis matrix** from $\mathcal{B}$ to $\mathcal{C}$. Accuracy in determining transition matrices, based on **Theorem 2.12** *Let $\mathcal{B} = (b_1, \ldots, b_n)$ and $\mathcal{C}$ be ordered bases for a vector space V. Then the change of basis operator $\phi_{\mathcal{B},\mathcal{C}} = \phi_{\mathcal{C}}\phi_{\mathcal{B}}^{-1}$ is an automorphism of F^n, whose standard matrix is* $M_{\mathcal{B},\mathcal{C}} = ([b_1]_{\mathcal{C}} \mid \cdots \mid [b_n]_{\mathcal{C}})$ *Hence* $[v]_{\mathcal{C}} = M_{\mathcal{B},\mathcal{C}}[v]_{\mathcal{B}}$ *and* $M_{\mathcal{C},\mathcal{B}} = M_{\mathcal{B},\mathcal{C}}^{-1}$ • Accuracy in determining two similar matrices			[1 x 3 x 50 minutes] (Certain conditions: Total *blended learning* : 50%)			WA group, learning video)		

13	An ability to understand /solve problems and properties in linear transformations	Accuracy in determining the matrix representation of a linear transformation with an ordered base defined in each vector space	Final exam: 6%	Teaching and discussions: Explanation of Learning Material [1 x 3 x 50 minutes]	Teaching and discussions: Explanation of Learning Material [1 x 3 x 50 minutes] (Certain conditions: Total *blended learning* : 50%)	Students read and study learning materials		PPT I learn (LMS Unand) • (Certain conditions: Zoom meeting, WA group, learning video)	Representation matrix	**6%**
14	An ability to solve problems and properties in quotient spaces and isomorphism theorems	• Accuracy in determining the mod quotient space , (is the vector space and is a subspace of $.VSVSV$ • Accuracy in proving	• Non Test Homework 9: 4% • Final exam : 3%	Teaching and discussions: Explanation of Learning Material Task Description [1 x 3 x 50 minutes]	Teaching and discussions: Explanation of Learning Material Task Description [1 x 3 x 50 minutes]	• Students read and study learning materials • Students do assignments independently		PPT I learn (LMS Unand) • (Certain conditions: Zoom meeting, WA group, learning video)	Quotient space	**7%**

		Theorem 3.1 *Let S be a subspace of V. The binary relation* $u \equiv v \Leftrightarrow u - v \in S$ *is an equivalence relation on V, whose equivalence classes are the* **cosets** $v + S = \{v + s \mid s \in S\}$ *of S in V. The set V/S of all cosets of S in V, called the* **quotient space** *of V modulo S, is a vector space under the well-defined operations* $r(u + S) = ru + S$ $(u + S) + (v + S) = (u + v) + S$ *The zero vector in V/S is the coset 0 + S = S.*□ **Theorem 3.2** *The canonical projection $\pi_S: V \to V/S$ defined by* $\pi_S(v) = v + S$ *is a surjective linear transformation with $\ker(\pi_S) = S$* □ **Theorem 3.3 (The correspondence theorem)** *the function that assigns to each intermedi subspace T/S of V/S is an order-preserving one-to-one correspondence between the set of a and the set of all subspaces of V/S.*			(Specific conditions: Total *blended learning* : 50%)					
15	An ability to solve problems and properties in quotient spaces and isomorphism theorems	• Accuracy in proving the universal nature of the quotient space • Accuracy in proving the main theorem of isomorphisms • Accuracy in proving **Theorem 3.4** *Let S be a subspace of V and let $\tau \in \mathcal{L}(V, W)$ satisfy $S \subseteq \ker(\tau)$. Then, as pictured in Figure 3.2, there is a unique linear transformation $\tau': V/S \to W$ with the property that* $\tau' \circ \pi_S = \tau$ *Moreover, $\ker(\tau') = \ker(\tau)/S$ and $\operatorname{im}(\tau') = \operatorname{im}(\tau)$.*	• Non Test Homework 10: 4% • Final exam : 3%	Teaching and discussions: Explanation of Learning Material Task Description [1 x 3 x 50 minutes]	Teaching and discussions: Explanation of Learning Material Task Description [1 x 3 x 50 minutes] (Specific conditions: Total *blended learning* : 50%)	• Students read and study learning materials • Students do assignments independently		PPT I learn (LMS Unand) • (Certain conditions: Zoom meeting, WA group, learning video)	The main theorem of isomorphisms	**7%**

		Theorem 3.4 *Let S be a subspace of V and let $\tau \in \mathcal{L}(V,W)$ satisfy $S \subseteq \ker(\tau)$. Then, as pictured in Figure 3.2, there is a unique linear transformation $\tau' : V/S \to W$ with the property that* $\tau' \circ \pi_S = \tau$ *Moreover, $\ker(\tau') = \ker(\tau)/S$ and $\operatorname{im}(\tau') = \operatorname{im}(\tau)$.*								
									Total Weight	100%
16	**FINAL EXAM**									

1 credit = 50 minutes face-to-face meeting, 60 minutes structured study, 60 minutes independent study
Each meeting duration is 3 credits = 3×50 minutes

Indicators, Criteria, and Assessment Weights

1. Assessment weight for each Assessment

NO	Assessment	Weight (%)
1	Mid-Term Exam	20
2	Final Exam	20
3	Homework	10
4	Final Project	50

TOTAL	100

2. Assessment weight for Intended Learning Outcome
   - CLO-1: 16 %
   - CLO-2: 20 %
   - CLO-3: 20 %
   - CLO- 4: 20 %
   - CLO-5: 20 %

Assessment Plan Table:

Task: 40%
Midterms: 30%
Final Semester Exam: 30%

Assessment Plan Table::

No.	Learning Outcomes	Weight (%)			
		Task (%)	Mid (%)	Final (%)	Total

1	Able to understand / solve problems and properties in vector spaces (CP2: P1-1, PI-2, PI-3, CP3: P1-1, PI-2, PI-3)	Task 1: 4 Task 2: 4 Task 3: 2	18		**28**
2	Able to understand / solve problems and properties in eigenvalues and eigenvectors (CP2: P1-1, PI-2, PI-3, CP3: P1-1, PI-2, PI-3)	Task 4: 5 Task 5: 5	12		**22**
3	Able to understand / solve problems and properties in linear transformations (CP2: P1-1, PI-2, PI-3, CP3: P1-1, PI-2, PI-3)	Task 6 : 4 (TM) Task 7: 4 (Kindergarten) Task 8 : 4 (TM)		16	**28**
4	Able to solve problems and properties in quotient spaces and isomorphism theorems (CP2: P1-1, PI-2, PI-3, CP3: P1-1, PI-2, PI-3)	Task 9: 4(TM) Task 10: 4 (Kindergarten)		14	**22**
	Total	**40**	**30**	**30**	**100**